Six-Field Governing Equation Development for Advanced System Codes

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ABSTRACT

System codes are used to analyze nuclear reactor systems during steady state and transient operations. These codes are able to predict pressure drop, void fraction distributions and temperature distributions for various coolants, heated flow geometries, and heat configurations. Advanced system codes are able to model two-phase flow regimes using built-in models and correlations to account for the heat transfer, drag, and interactions between phases. In spite of this, extreme flow conditions can tax current code capabilities, particularly those flows that involve significant phase change. Current system codes (such as RELAP5-3D, RELAP-7, and TRACE) have a six-equation model that includes mass, momentum, and energy conservation equations for two fields (liquid and vapor).

Work on the conservation equations in a few of these codes has included development of a separate droplet field from the continuous liquid. This is in line with a trend toward the inclusion of more fields in system codes. The representation of two phase flow phenomena is improved by increasing the number of fields. Conservation equations based on six fields (liquid, vapor, small bubble, large bubble, small droplet and large droplet) are derived in this work.

KEYWORDS
two-phase flow, new fields, multi-field, conservation equation

1. INTRODUCTION

Nuclear reactor systems are complex, and require detailed analysis to evaluate reactor performance during normal operations as well as accident or transient conditions. Computer codes that are used to analyze these complex reactor systems are called "system codes".

These hydrodynamic models have frequently been extended to include a code capability to model multiple phase flows [1, 2]. The interaction between phases in the coolant is modeled in order to capture heat transfer properties and mass exchange between the phases.

Conservation equations are used to balance the mass, momentum, and energy within a control volume or phase. Mass, momentum, and energy balances are computed to account for convective effects, heat added to or removed from the control volume or phase, and other characteristics such as energy loss to diffusion or viscous effects. For two phase systems, mass, momentum, and energy may also be exchanged by a change in phase. Complete characterization of a phase requires an equation for the mass, momentum, and energy balance, along with the closure relationships for that phase.

Generally, the code models include just two fields, one for each phase. Such a model is limited to capturing the characteristics of a liquid and vapor by using the lumped capacitance approximation. This approximation applies to two fields by assuming that all the liquid (continuous liquid and droplets) are only one field having the same temperature, pressure, and velocity. The same approximation applies to the vapor field, where the continuous vapor and the bubbles are both covered by a single field and share a single velocity, temperature, and pressure.

Some nuclear reactor designs, in particular Boiling Water Reactors (BWRs), operate at steady state with coolant that ranges from subcooled liquid to saturated steam. Other reactor designs can experience severe accident scenarios (such as core reflood, blowdown, or rapid depressurization) where rapid and extreme changes in coolant vapor content will tax the capabilities of a two-field model. The steady state BWR conditions and severe accident scenarios can involve bubbles and droplets of varying size. Reactor system characteristics and accident progress are affected by the heat transfer between these additional fields.

More complex models increase the number of fields, including liquid droplets or bubbles as additional fields [1]. As with the inclusion of additional phases, each field requires additional conservation equations and closure relationships to be modeled effectively by the code.

Further progress in system codes is expected to come from multifield modeling [3]. The current trends in system code development include the improvement of the two phase models by increasing the number of fields. This paper will show the development of two-phase, six-field conservation equations for the following six fields:

1. Continuous Liquid
2. Continuous Vapor

} Fields currently included in many system codes [1, 2]

3. Large Bubble
4. Small Bubble
5. Large Droplet
6. Small Droplet

} Four proposed fields

2. SIX-FIELD CONSERVATION EQUATIONS

Reference [4] provides the derivation of a generalized conservation equation for a general property of phase (or field) k. The complete set of conservation equations in mass, momentum, and energy is presented for each of the six proposed fields, beginning with mass conservation.

2.1. Mass Continuity

The mass conservation equation is:

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k) = \Gamma_k \tag{1}$$

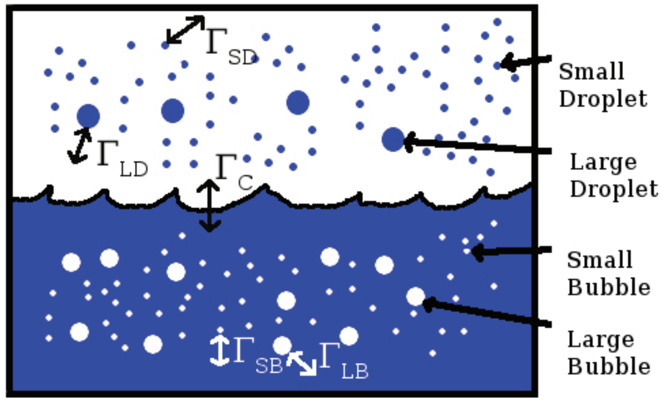

Figure 1: Phasic interaction examples

Note that the variables in the governing equations presented here are averaged in space and time.

The two-field model formulation used in most system codes includes a single interface for the transfer of mass and energy between the phases/fields. Figure 1 provides a graphical depiction of the mass transfer between the proposed fields in an arbitrary volume.

The figure shows mass exchange due to phase exchange between the continuous vapor and the large (Γ_{LD}) and small (Γ_{SD}) droplets, as well as between the continuous liquid and the large and small bubbles (Γ_{LB}, Γ_{SBu}) and the continuous liquid to the continuous vapor (Γ_C). The mass exchange due to phase change can be in the form of evaporation (vapor generation) or condensation. The vapor mass generation is equal in magnitude, but opposite in sign from the liquid mass generation.

There are five interfaces between the fields where phase change will result in mass exchange between the phases. The total vapor generation from all liquid fields is represented by Γ_g. It is assumed that the total vapor generation can be partitioned into mass transfer at the vapor/liquid interface (Γ_{ig}), and that in the thermal boundary layer near the walls (Γ_w) [5].

The net heat and mass transfer across the phase interface is assumed to be zero; this is because the interface is assumed to have no volume. Thus, all the energy or mass leaving one phase must cross the interface into the other phase. The following terms are used to represent the vapor generation rate from the continuous liquid and droplet fields.

Γ_L = Vapor generation rate from continuous liquid

Γ_{LD} = Vapor generation rate from large droplets

Γ_{SD} = Vapor generation rate from small droplets

In addition to mass transfer from phase changes, mass can be transferred from one field to another by physical mechanisms. This includes large droplets impacting fuel rod spacer grids and breaking up into smaller droplets, droplets being entrained in the vapor from the continuous liquid due to high relative velocities between the phases or vigorous bubble generation due to heating, and other mechanisms.

These physical mechanisms are included in the mass continuity equation in the form of source terms.

Specific closure models are needed to provide the values of these terms in system analyses.

Based on the above considerations, mass conservation equations can be developed for each of the considered fields. Equation 2 shows the mass conservation for the continuous liquid. Note the two additional terms for the small and large droplet fields. It is assumed that there is no direct droplet-to-bubble phase transfer, so no terms are included representing that mode.

$$\underbrace{\frac{\partial}{\partial t}(\alpha_f \rho_f)}_{\text{A}} + \underbrace{\nabla\cdot(\alpha_f \rho_f \vec{v}_f)}_{\text{B}} = \underbrace{-\Gamma_L}_{\text{C}} - \underbrace{S'''_{LD,E}}_{\text{D}} - \underbrace{S'''_{SD,E}}_{\text{E}} + \underbrace{S'''_{LD,DE}}_{\text{F}} + \underbrace{S'''_{SD,DE}}_{\text{G}} \tag{2}$$

A - Rate of change of mass

B - Mass change due to convection

C - Rate of mass transfer from continuous liquid due to phase change

D - Mass lost to large droplet field by entrainment

E - Mass lost to small droplet field by entrainment

F - Mass gained from large droplet field by de-entrainment

G - Mass gained from small droplet field by de-entrainment

The mass conservation for large droplet fields is shown as Equation 3. Equation 4 shows the same mass conservation equation for small droplets.

$$\frac{\partial}{\partial t}(\alpha_{LD} \rho_f) + \nabla\cdot(\alpha_{LD} \rho_f \vec{v}_{LD}) = \underbrace{-\Gamma_{LD}}_{\text{A}} + S'''_{LD,E} - S'''_{LD,DE} - \underbrace{S'''_{LD,SB}}_{\text{B}} - \underbrace{S'''_{LD,FB}}_{\text{C}} + \underbrace{S'''_{SD,C}}_{\text{D}} \tag{3}$$

A - Mass lost due to large droplet evaporation

B - Mass lost to small droplet by spacer-breakup

C - Mass lost to small droplet field by flow breakup

D - Mass gained from small droplet field coalescence

$$\frac{\partial}{\partial t}(\alpha_{SD} \rho_f) + \nabla\cdot(\alpha_{SD} \rho_f \vec{v}_{SD}) = \underbrace{-\Gamma_{SD}}_{\text{A}} + S'''_{SD,E} + S'''_{LD,SB} + S'''_{LD,FB} - S'''_{SD,DE} - S'''_{SD,C} \tag{4}$$

A - Mass lost due to small droplet evaporation

The following three equations were developed following the same pattern used for the liquid phases. Equation 5 is the mass balance for the continuous vapor phase. This equation and those following include additional terms for the mass exchange between the new fields (large and small bubbles).

$$\frac{\partial}{\partial t}\left(\alpha_g \rho_g\right) + \nabla\cdot\left(\alpha_g \rho_g \vec{v}_g\right) = \underbrace{\Gamma_{CV}}_{\text{A}} - \underbrace{S^{'''}_{SBu,E}}_{\text{B}} + \underbrace{S^{'''}_{LB,C}}_{\text{C}} - \underbrace{S^{'''}_{LB,E}}_{\text{D}} + \underbrace{S^{'''}_{LB,DE}}_{\text{E}} + \underbrace{S^{'''}_{SB,DE}}_{\text{F}} \tag{5}$$

A - Rate of mass gained by continuous vapor from phase change

B - Rate of mass loss by entrainment of small bubbles

C - Rate of mass gain by coalescence of large bubbles

D - Rate of mass loss by entrainment of large bubbles

E - Rate of mass gain by de-entrainment of large bubbles

F - Rate of mass gain by de-entrainment of small bubbles

Note from equation 5 that there is no term for the gain of mass in the continuous vapor field due to the coalescence of small bubbles. This formulation assumes that the small bubbles must first coalesce into large bubbles before they will coalesce to the point of being considered part of the continuous vapor field. Thus, the rate of mass gain by coalescence of small bubbles is not needed. The next two equations (6, 7) are for mass conservation in the new large bubble and small bubble fields, respectively.

$$\frac{\partial}{\partial t}\left(\alpha_{LB} \rho_g\right) + \nabla\cdot\left(\alpha_{LB} \rho_g \vec{v}_{LB}\right) = \underbrace{\Gamma_{LB}}_{\text{A}} + \underbrace{S^{'''}_{SBu,C}}_{\text{B}} - S^{'''}_{LB,C} + S^{'''}_{LB,E} - S^{'''}_{LB,DE} - \underbrace{S^{'''}_{LB,SB}}_{\text{C}} - \underbrace{S^{'''}_{LB,FB}}_{\text{D}} \tag{6}$$

A - Rate of large bubble generation due to evaporation

B - Mass gained by large bubbles by small bubble coalescence

C - Mass lost from large bubble field by spacer grid breakup

D - Mass lost from large bubble field by flow breakup

$$\frac{\partial}{\partial t}\left(\alpha_{SBu}\rho_g\right) + \nabla\cdot\left(\alpha_{SBu}\rho_g\vec{v}_{SBu}\right) = \underbrace{\Gamma_{SBu}}_{\text{A}} + S'''_{SBu,E} - S'''_{SBu,DE} - S'''_{SBu,C} + S'''_{LB,SB} + S'''_{LB,FB} \tag{7}$$

A - Rate of small bubble generation due to evaporation

2.2. Momentum Continuity

Interactions between phases are not limited to mass exchange. In fact, significant importance should be applied to the exchange of momentum between phases. Any phase interface will involve the exchange of momentum. This momentum transfer is the reason that droplets of water are carried along in a vapor flow. High velocity vapor flows are one of the primary sources of droplets in the vapor field (by way of entrainment).

Momentum equations are now derived for each field. With a separate equation for each field, the droplets can have different velocities than the continuous liquid, as well as different velocities from one another. The same is true for the large and small bubble fields. Closure relationships for the physical models are required to solve the momentum equations by providing the details of the mass and momentum exchange between the fields and the drag forces at the walls. The closure models provide information about the turbulence and entrainment for the continuous fields.

The momentum conservation equation for continuous liquid, including the terms for momentum exchange with the additional fields is shown as Equation 8. Note the additional terms for momentum exchange due to droplet entrainment/de-entrainment, and that the variables are averaged.

$$\begin{aligned}
&\underbrace{\alpha_f\rho_f\frac{D\vec{v}_f}{Dt}}_{\text{A}} = \underbrace{-\alpha_f\nabla p_f}_{\text{B}} + \underbrace{\nabla\cdot\left[\alpha_f\left(\mathfrak{T}_f + \mathfrak{T}_f^T\right)\right]}_{\text{C}} + \underbrace{\alpha_f\rho_f\vec{g}_f}_{\text{D}} + \\
&\underbrace{\left(p_{fi} - p_f\right)\nabla\alpha_f}_{\text{E}} + \underbrace{\left(\vec{v}_{i,L} - \vec{v}_f\right)\Gamma_L}_{\text{F}} + \underbrace{\left(\vec{v}_{i,LB} - \vec{v}_f\right)\Gamma_{LB}}_{\text{G}} + \\
&\underbrace{\left(\vec{v}_{i,SBu} - \vec{v}_f\right)\Gamma_{SBu}}_{\text{H}} + \underbrace{M_{if}}_{\text{I}} - \underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{fi,g}}_{\text{J}} - \underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{fi,SBu}}_{\text{K}} - \\
&\underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{fi,LB}}_{\text{L}} - \underbrace{S'''_{LD,E}v_{LD}}_{\text{M}} - \underbrace{S'''_{SD,E}v_{SD}}_{\text{N}} + \underbrace{S'''_{SD,DE}v_{SD}}_{\text{O}} + \\
&\underbrace{S'''_{LD,DE}v_{LD}}_{\text{P}}
\end{aligned} \tag{8}$$

Where the equation terms are defined in words:

A - Rate of change of liquid momentum, including convective effects

B - Pressure gradient in continuous liquid

C - Momentum change from average viscous stress and the turbulent stress effects

D - Momentum change from body forces (gravity, pump head)

E - Pressure difference between interface and continuous liquid

F - Momentum change due to phase change across the continuous liquid/vapor interface

G - Momentum change due to phase change across the large bubble/vapor interface

H - Momentum change due to phase change across the small bubble/vapor interface

I - Interfacial drag from pressure imbalance at interface and skin drag from imbalanced shear forces

J - Momentum lost due to average interfacial shear stress at continuous vapor interface

K - Momentum lost due to average interfacial shear stress at small bubble interface

L - Momentum lost due to average interfacial shear stress at large bubble interface

M - Momentum lost due to large droplet entrainment

N - Momentum lost due to small droplet entrainment

O - Momentum gained from small droplet de-entrainment (to continuous liquid field)

P - Momentum gained from large droplet de-entrainment

Equation 9 is for momentum conservation in the small droplet field. In addition to the entrainment and de-entrainment terms from Equation 8, Equation 9 includes terms for momentum increase due to the breakup of large droplets, from grid spacers or flow effects. Note that the viscous and turbulent stress effects on momentum are not included in the small droplet momentum conservation equation. It is assumed that these effects only affect the fluid within the droplet itself, and that as such, the effects do not impact the general balance equations. Interfacial effects between the droplets and the vapor field are captured elsewhere in the balance equation.

$$\begin{aligned}\alpha_{SD}\rho_f\frac{D\vec{v}_{SD}}{Dt} &= -\alpha_{SD}\nabla p_{SD} + \alpha_{SD}\rho_f\vec{g}_{SD} + \left(p_{i,SD} - p_{SD}\right)\nabla\alpha_{SD} + \\ &\quad \left(\vec{v}_{i,SD} - \vec{v}_{SD}\right)\Gamma_{SD} + M_{i,SD} - \nabla\alpha_{SD}\cdot\mathfrak{T}_{SDi,g} + \\ &\underbrace{S'''_{SD,E}v_{SD}}_{A} + \underbrace{S'''_{LD,SB}v_{LD}}_{B} + \underbrace{S'''_{LD,FB}v_{LD}}_{C} - \underbrace{S'''_{SD,DE}v_{SD}}_{D} - \underbrace{S'''_{SD,C}v_{SD}}_{E}\end{aligned} \tag{9}$$

A - Momentum lost due to small droplet entrainment

B - Momentum increase from spacer breakup of large droplets

C - Momentum increase from flowing breakup of large droplets

D - Momentum lost from small droplet de-entrainment

E - Momentum lost from small droplet coalescence (to large droplets)

The large droplet momentum balance is similar to that for the small droplet balance and is shown in Equation 10. The terms of the equation are the same as those in the equations previously presented, but are for large droplets.

$$\alpha_{LD}\rho_f\frac{D\vec{v}_{LD}}{Dt} = -\alpha_{LD}\nabla p_{LD} + \alpha_{LD}\rho_f\vec{g}_{LD} + (p_{i,LD} - p_{LD})\nabla\alpha_{LD} + (\vec{v}_{i,LD} - \vec{v}_{LD})\Gamma_{LD} + M_{i,LD} - \nabla\alpha_{LD}\cdot\mathfrak{T}_{LDi,g} + \underbrace{S'''_{LD,E}v_{LD}}_{A} - \underbrace{S'''_{LD,SB}v_{LD}}_{B} - \underbrace{S'''_{LD,FB}v_{LD}}_{C} - \underbrace{S'''_{LD,DE}v_{LD}}_{D} + \underbrace{S'''_{SD,C}v_{SD}}_{E} \tag{10}$$

The momentum balance in the continuous vapor field is given by Equation 11. The terms are very similar to those for the continuous liquid equation.

$$\alpha_g\rho_g\frac{D\vec{v}_g}{Dt} = -\alpha_g\nabla p_g + \nabla\cdot\left[\alpha_g\left(\mathfrak{T}_g + \mathfrak{T}_g^T\right)\right] + \alpha_g\rho_g\vec{g}_g + (p_{gi} - p_g)\nabla\alpha_g + (\vec{v}_{i,g} - \vec{v}_g)\Gamma_g + (\vec{v}_{i,LD} - \vec{v}_g)\Gamma_{LD} + (\vec{v}_{i,SD} - \vec{v}_g)\Gamma_{SD} + M_{ig} - \underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{gi,f}}_{A} - \underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{gi,SD}}_{B} - \underbrace{\nabla\alpha_f\cdot\mathfrak{T}_{gi,LD}}_{C} + \underbrace{\Gamma_{LD}(v_{LD,i} - v_{LD})}_{D} + \underbrace{\Gamma_{SD}(v_{SD,i} - v_{SD})}_{E} \tag{11}$$

A - Momentum loss due to average interfacial shear stress at continuous liquid interface

B - Momentum loss to interfacial shear at the small droplet interface

C - Momentum loss to interfacial shear at the large droplet interface

D - Momentum increase from large droplet evaporation

E - Momentum increase from small droplet evaporation

The small and large bubble momentum conservation equations are shown in Equations 12 and 13. The bubbles are also susceptible to entrainment and de-entrainment by coalescence and breakup. The additional terms for these effects are indicated for each equation.

$$\alpha_{SBu}\rho_g\frac{D\vec{v}_{SBu}}{Dt} = -\alpha_{SBu}\nabla p_{SBu} + \alpha_{SBu}\rho_g\vec{g}_{SBu} + (p_{i,SBu} - p_{SBu})\nabla\alpha_{SBu} + (\vec{v}_{i,SB} - \vec{v}_{SBu})\Gamma_{SB} + M_{i,SBu} - \nabla\alpha_{SBu}\cdot\mathfrak{T}_{SBi,f} + \underbrace{S'''_{SB,E}v_{SBu}}_{A} + \underbrace{S'''_{LB,SB}v_{LB}}_{B} + \underbrace{S'''_{LB,FB}v_{LB}}_{C} - \underbrace{S'''_{SB,DE}v_{SBu}}_{D} - \underbrace{S'''_{SB,C}v_{SBu}}_{E} \tag{12}$$

A - Momentum change due to small bubble entrainment

B - Momentum increase due to large bubble spacer grid breakup

C - Momentum increase due to large bubble flow-driven breakup

D - Momentum lost to continuous vapor by small bubble de-entrainment

E - Momentum lost to large bubbles by small bubble coalescence

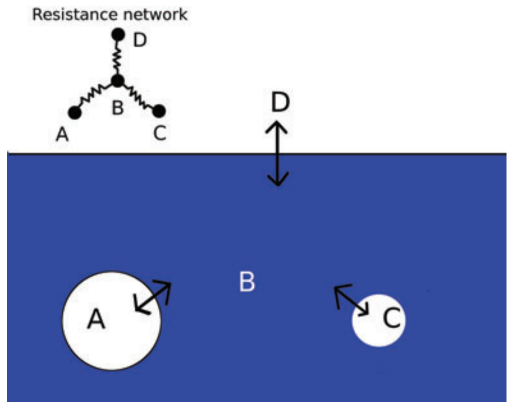

Figure 2: Bubble to liquid heat transfer

$$\alpha_{LB}\rho_g\frac{D\vec{v}_{LB}}{Dt} = -\alpha_{LB}\nabla p_{LB} + \alpha_{LB}\rho_g\vec{g}_{LB} + \left(p_{i,LB} - p_{LB}\right)\nabla\alpha_{LB} + \left(\vec{v}_{i,LB} - \vec{v}_{LB}\right)\Gamma_{LB} + M_{i,LB} - \nabla\alpha_{LB}\cdot\mathfrak{T}_{LBi,f} + \underbrace{S'''_{LB,E}v_{LB}}_{A} - S'''_{LB,SB}v_{LB} - S'''_{LB,FB}v_{LB} - \underbrace{S'''_{LB,DE}v_{LB}}_{B} + S'''_{SB,C}v_{SBu} \tag{13}$$

A - Momentum change due to large bubble entrainment

B - Momentum lost to continuous vapor by large bubble de-entrainment

2.3. Energy Continuity

The details of the derivation of the energy governing equations are provided in [4]. The turbulent energy source (Φ_k^T) and the viscous dissipation term (Φ_k^μ) are defined below[6]:

$$\Phi_k^T \equiv -\widehat{\vec{v}_k}\cdot\nabla\cdot\left(\alpha_k\mathfrak{T}_k^T\right) + W_{ki}^T \tag{14}$$

$$\Phi_k^\mu \equiv \alpha_k\overline{\overline{\mathfrak{T}_k}} : \nabla\widehat{\vec{v}_k} \tag{15}$$

The current development of the energy balance equations includes a separate equation for each field and the additional applicable terms for energy exchange between them. The primary mechanism for energy exchange between the continuous liquid and the droplets is from direct droplet entrainment.

The direct heat transfer between the continuous vapor and the vapor bubbles, as well as between the large and small bubbles can be neglected, since the heat transfer between those fields is included in the bubble to continuous liquid and the continuous liquid to vapor (or bubble) heat transfer. The continuous liquid phase becomes the medium for the heat transfer between the bubble fields and between a bubble field and the vapor field. Figure 2 shows this exchange graphically.

If we assume that the temperature in Bubble A is greater than the temperature of the coolant (point B), there will be heat transfer from the surface of bubble A to the continuous liquid at B. The heat transfer through the interface between bubble A and the continuous liquid is represented as a resistor in the nodal network shown as an inset to Figure 2. As heat is transferred to the liquid, the temperature will rise. If the temperature in the continuous liquid (point B) is higher than the continuous vapor (point D) or a different (small) bubble (point C), the heat may subsequently be transferred to those fields.

The remaining heat exchange modes between the fields, and those that must be captured by the conservation equations are:

- Continuous liquid to/from vapor
- Continuous liquid to/from droplets by droplet entrainment/de-entrainment
- Continuous liquid to/from bubbles
- Continuous vapor to/from droplets
- Continuous vapor to/from bubbles by bubble entrainment/de-entrainment

Closure relationships will be required to capture the heat transfer between the fields. If appropriate closure relationships can not be defined, the droplet and liquid temperatures may be assumed to be the same to reduce the required information for the energy balance. The energy balance for the continuous liquid field is shown as Equation 16. Note that the terms in the equations are averaged as with the other equations.

$$\begin{aligned}
\underbrace{\alpha_f \rho_f \frac{D_f h_f}{Dt}}_{A} &= \underbrace{-\nabla\cdot \alpha_f \left(\vec{q}_f + \vec{q}_f^T\right)}_{B} \underbrace{+\alpha_f \frac{D_f p_f}{Dt}}_{C} + \Phi_f^T + \Phi_f^\mu + \\
&\underbrace{\Gamma_{f,i}\left(h_{f,i} - h_f\right)}_{D} + \underbrace{\Gamma_{f,w}\left(h_{f,w} - h_f\right)}_{E} + \underbrace{\Gamma_{f,SBu}\left(h_{f,SBu} - h_f\right)}_{F} + \\
&\underbrace{\Gamma_{f,LB}\left(h_{f,LB} - h_f\right)}_{G} + \underbrace{a_i q''_{f,i}}_{H} + \underbrace{a_{i,SBu} q''_{SBu,i}}_{I} + \underbrace{a_{i,LB} q''_{LB,i}}_{J} + \underbrace{a_{w,f} q''_{w,f}}_{K} + \\
&\underbrace{\left(p_f - p_{f,i}\right)\frac{D_f \alpha_f}{Dt}}_{L} + \underbrace{M_{i,f}\cdot\left(\vec{v}_{f,i} - \vec{v}_f\right)}_{M} - \underbrace{\nabla\alpha_f\cdot \mathfrak{T}_{f,i}\cdot\left(\vec{v}_{f,i} - \vec{v}_f\right)}_{N} - \\
&S'''_{LD,E} h_f - S'''_{SD,E} h_f + S'''_{LD,DE} h_{LD} + S'''_{SD,DE} h_{SD}
\end{aligned} \tag{16}$$

Where the terms are defined as:

A - Time rate of change of energy in terms of enthalpy, including convective effects

B - Energy transfer from average conduction and turbulent heat flux

C - Energy from averaged flow work term

D - Energy transfer due to phase change at the continuous vapor interface

E - Energy transfer due to phase change at the wall

F - Energy transfer due to phase change at the small bubble interface

G - Energy transfer due to phase change at the large bubble interface

H - Energy transferred between continuous liquid and continuous vapor fields

I - Energy transferred between continuous liquid and small bubbles

J - Energy transferred between continuous liquid and large bubbles

K - Energy transferred from the wall to the continuous liquid

L - Energy transfer from pressure differences at interface

M - Energy transferred due to interfacial drag between continuous fields

N - Energy transfer from interfacial shear stress

Equation 17 is the energy conservation equation for small droplets, while Equation 18 is for large droplets.

$$\begin{aligned}\alpha_{SD}\rho_{SD}\frac{D_{SD}h_{SD}}{Dt} &= \Gamma_{SD,i}\left(h_{SD,i}-h_{SD}\right)+a_i q''_{SD,i}+\\(p_{SD}-p_{SD,i})\frac{D_{SD}\alpha_{SD}}{Dt} &+ M_{i,SD}\cdot\left(\vec{v}_{SD,i}-\vec{v}_{SD}\right)-\\\nabla\alpha_{SD}\cdot\mathfrak{T}_{SD,i}\cdot\left(\vec{v}_{SD,i}-\vec{v}_{SD}\right) &- S'''_{LD,C}h_{SD}+S'''_{SD,E}h_f+\\S'''_{LD,SB}h_{LD} &+ S'''_{LD,FB}h_{LD}-S'''_{SD,DE}h_{SD}\end{aligned} \tag{17}$$

$$\begin{aligned}\alpha_{LD}\rho_{LD}\frac{D_{LD}h_{LD}}{Dt} &= \Gamma_{LD,i}\left(h_{LD,i}-h_{LD}\right)+a_i q''_{LD,i}+\\(p_{LD}-p_{LD,i})\frac{D_{LD}\alpha_{LD}}{Dt} &+ M_{i,LD}\cdot\left(\vec{v}_{LD,i}-\vec{v}_{LD}\right)-\\\nabla\alpha_{LD}\cdot\mathfrak{T}_{LD,i}\cdot\left(\vec{v}_{LD,i}-\vec{v}_{LD}\right) &+ S'''_{LD,C}h_{SD}+S'''_{LD,E}h_f-\\S'''_{LD,SB}h_{LD} &- S'''_{LD,FB}h_{LD}-S'''_{LD,DE}h_{LD}\end{aligned} \tag{18}$$

Note the presence of the entrainment/de-entrainment source terms in Equation 17 and Equation 18. These source terms (and similar terms in the equations for energy conservation in the bubble fields) are dependent to some extent on the flow phenomena and the geometry of the flow. Improved model performance will be possible if closure relationships are used that compensate for variations in the flow characteristics. The energy balance for the continuous vapor field is shown in Equation 19.

$$\begin{aligned}\alpha_g\rho_g\frac{D_g h_g}{Dt} &= -\nabla\cdot\alpha_g\left(\vec{q}_g+\vec{q}_g^{\,T}\right)+\alpha_g\frac{D_g p_g}{Dt}+\Phi_g^T+\Phi_g^\mu+\\\Gamma_{g,i}\left(h_{f,i}-h_f\right) &+ \Gamma_{g,w}\left(h_{f,w}-h_f\right)+\Gamma_{g,SBu}\left(h_{f,SBu}-h_f\right)+\\\Gamma_{g,LB}\left(h_{f,LB}-h_f\right)+a_i q''_{g,i} &+ a_{g,SD}q''_{SD,i}+a_{i,LD}q''_{LD,i}+a_{w,g}q''_{w,g}+\\(p_g-p_{g,i})\frac{D_g\alpha_g}{Dt} &+ M_{i,g}\cdot\left(\vec{v}_{g,i}-\vec{v}_g\right)-\nabla\alpha_g\cdot\mathfrak{T}_{g,i}\cdot\left(\vec{v}_{g,i}-\vec{v}_g\right)-\\S'''_{LB,E}h_g &- S'''_{SB,E}h_g+S'''_{LB,DE}h_{LB}+S'''_{SB,DE}h_{SBu}\end{aligned} \tag{19}$$

The equation below is the energy conservation for small bubbles. Equation 21 is the energy conservation equation for large bubbles.

$$\begin{aligned}
\alpha_{SBu}\rho_{SBu}\frac{D_{SBu}h_{SBu}}{Dt} = &\underbrace{\Gamma_{SBu,i}\left(h_{SBu,i} - h_{SBu}\right)}_{\text{A}} + a_i q''_{SBu,i} + \\
&\left(p_{SBu} - p_{SBu,i}\right)\frac{D_{SBu}\alpha_{SBu}}{Dt} + M_{i,SBu}\cdot\left(\vec{v}_{SBu,i} - \vec{v}_{SBu}\right) - \\
&\nabla\alpha_{SBu}\cdot\mathfrak{T}_{SBu,i}\cdot\left(\vec{v}_{SBu,i} - \vec{v}_{SBu}\right) - S'''_{SBu,C}h_{SBu} + S'''_{SBu,E}h_g + \\
&S'''_{LB,SB}h_{LB} + S'''_{LB,FB}h_{LB} - S'''_{SBu,DE}h_{SBu}
\end{aligned} \tag{20}$$

A - Phase change from small bubbles to liquid

$$\begin{aligned}
\alpha_{LB}\rho_{LB}\frac{D_{LB}h_{LB}}{Dt} = &\Gamma_{LB,i}\left(h_{LB,i} - h_{LB}\right) + a_i q''_{LB,i} + \\
&\left(p_{LB} - p_{LB,i}\right)\frac{D_{LB}\alpha_{LB}}{Dt} + M_{i,LB}\cdot\left(\vec{v}_{LB,i} - \vec{v}_{LB}\right) - \\
&\nabla\alpha_{LB}\cdot\mathfrak{T}_{LB,i}\cdot\left(\vec{v}_{LB,i} - \vec{v}_{LB}\right) + S'''_{SBu,C}h_{SBu} + S'''_{LB,E}h_f - \\
&S'''_{LB,SB}h_{LB} - S'''_{LB,FB}h_{LB} - S'''_{LB,DE}h_{LB}
\end{aligned} \tag{21}$$

3. CONCLUSION

Current system codes typically have only two fields, one for each phase (liquid and vapor). This simplification assumes that all the liquid (continuous liquid, large droplets, and small droplets) are modeled as one field having the same temperature, pressure, and velocity. The same approximation applies to the vapor field, where the continuous vapor as well as the large and small bubbles are represented by a single field and share a single velocity, temperature, and pressure.

Severe accident scenarios tax the capabilities of a two-field model. Many of the modern system codes are being expanded to include conservation equations to model additional fields. These changes improve the modeling capabilities of these codes.

The two-phase six-field model derived herein includes six mass conservation equations that include source terms that represent the mass transfer between six fields: liquid, small droplet, large droplet, vapor, small bubble, and large bubble. Mass transfer between fields can occur due to phase change and physical mechanisms. The physical mechanisms include liquid and droplets interacting with the spacer grids and high velocity vapor flows causing droplet entrainment. Specific closure relationships are required to provide the value of these source terms in a full system analysis.

The six momentum continuity equations have been developed by including momentum exchange resulting from phase change and the same physical mechanisms that affect the mass continuity. The current derivation is formulated to allow for the droplet fields to have different velocities from the continuous liquid, as well as different velocities for the two droplet fields.

The energy conservation equations presented here are structured to track separate temperatures for the droplet fields and the continuous liquid field.

The six field model has been developed to include the effects of flows and interfacial exchange for six fields in two-phase flows. Inclusion of additional fields is expected to improve model performance for BWR reactors and accident conditions in other reactor designs.

Nomenclature

Greek

η Fraction of vapor generation from large and small droplet fields

Γ Interfacial phasic mass transfer

Γ_g Average rate of vapor generation per unit volume from all liquid fields

Φ_k^{μ} Viscous dissipation term

Φ_k^T Turbulent work effect source

English

$q_{SBu,i}$ heat lost from small bubbles to liquid

$S'''_{LB,DE}h_{LB}$ large bubble de-entrainment to vapor field

$S'''_{LB,FB}h_{LB}$ large bubble breakup from flow

$S'''_{LB,SB}h_{LB}$ large bubble breakup from spacer grid

$S'''_{SBu,C}h_{SBu}$ small bubble to large bubble field (from coalescence)

$S'''_{SBu,DE}h_{SBu}$ small bubble de-entrainment

$S'''_{SBu,E}h_g$ small bubble entrainment from cont. vapor

$S'''_{SD,E}h_f$ small droplet entrainment from cont. liquid

Subscripts

C Coalescence

CV Continuous vapor

DE De-entrainment

E Entrainment

f Liquid

FB Flow-caused breakup

g Vapor

I Interface

L Continuous liquid field

LB Large bubble

LD Large droplet field

SB	Spacer-caused breakup
SBu	Small bubble
SD	Small droplet field

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