A NEW LUMPED THERMAL RESISTANCE HEAT TRANSFER MODEL FOR FUEL PIN STRUCTURE

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ABSTRACT

Lumped parameter heat transfer methodology is simple, and the solution is very fast, so the lumped parameter approach has been widely used in the thermal-hydraulic analysis for the fuel pin heat transfer in the nuclear reactors. In the conventional lumped parameter thermal analysis of the fuel pin structure, each component (such as pellet, cladding etc.) is characterized by a concentrated bulk temperature (or averaged temperature), and a bulk thermal resistance. In contrast to this conventional lumped thermal resistance model, in this paper another kind of lumped thermal resistance heat transfer model for fuel pin structure has been developed. In this model, each fuel pin component is still represented by a concentrated lumped mean temperature, while the location of the mean temperature position of each component is no longer set on the geometrical middle point, rather exactly assigned on the analytical temperature profile. Two thermal resistance elements are assigned for each component in this new model: between each component outer surface and lumped mean temperature node a thermal resistance is assigned, respectively, each lumped thermal resistance connects the mean temperature node with the corresponding outer surface. The heat conduction between mean temperature nodes of different components is properly defined to takes place at the in-between surfaces. Within this new model, the location of the mean temperature positions for each component can be determined analytically, and all the thermal resistances are re-defined, accordingly. The advantage of the presented method is that the temperature profile in the fuel structure at any radial position can be re-produced after a quite easily lumped heat transfer calculation. This methodology can be used in nuclear reactor simulation studies where fastness of the solution is a matter of concern, meanwhile the exact temperature profile in the fuel pin structure can be re-produced at the same time.

KEYWORDS
Fuel pin heat transfer; Lumped parameter thermal resistance;
Fuel pin mean temperature; Temperature profile in the fuel pin structure.

1. INTRODUCTION

Solving the thermal problems with the exact analytical method or numerical analysis sometimes is cumbersome or expensive in terms of the computing time. Under some conditions, an alternative approach, based on the lumped parameter method, becomes attractive and feasible. Especially for nuclear reactor analysis, the lumped parameter method is very useful because the set of governing partial differential equations is reduced to a few algebraic equations that can be easily solved with less effort. This method becomes more attractive, if the physical model of the system is very complicated. Thus

lumped parameter heat transfer models have been widely used in the thermo-hydraulic analysis of nuclear reactors.

In some nuclear reactor analysis applications, our interesting focus on the "steady" temperature distribution within fuel pin structure under the assumption that pasting the initial transient phase the temperature distribution reaches its time asymptotic solution, in this circumstance the lumped parameter, combined with circuit analogy method, is of great advantage because of its simplicity. The thermal resistance concept permits some relatively complex heat-transfer problems to be solved in a very simple manner.

The conventional lumped parameter thermal analysis applies only to heat transfer problems without internal energy generation. [1, 2], because of the assumption that the heat fluxes keep constant in the analyzed components. As an approximation, the distributed heat generation source in one component can also be concentrated on its geometrical central position. Meanwhile this point services also for the location of the mean temperature positon within the component. This method is widely applied in equivalent thermal network analysis for electrical machines. [3, 4].

On the contrary, in this paper another kind of lumped-parameter thermal resistance is developed for fuel pin structure with central hole or without central hole, under the assumption of a uniform volumetric internal heat source. This thermal resistance concept can exactly reflect the influence of the internal heat source without any approximation.

The goal of this paper is to find a general and suitable lumped parameter thermal resistance model for a fuel pin structure. The treatment has been generalized by considering the heat transfers between the surfaces of the considered component and the mean temperature location, see Fig. 1. This lumped parameter thermal resistance concept is very general, and it can also be deployed to the commonly used thermal network approach for other engineering applications, such as the analysis of electric machines, etc. [5].

2. BASIC EQUATIONS AND MATHEMATICAL MODELS

2.1 Temperature Profile and Heat Transfer Rate

Consider a hollow cylindrical pellet of inner and outer radii R_i and R_o, respectively, with a uniform nuclear heat generation rate $\dot{q}$ per unit volume. The length of the hollow cylindrical pellet is assumed infinitely and material thermal properties are axially homogenous, see Fig. 1.

The 2D cylindrical coordinate system is employed in the following analysis. The thermal conductivity, k, fuel mass density, ρ, and the specific heat capacity, c, the nuclear heat generation rate, $\dot{q}$, per unit volume are assumed constant.

It is well known that one dimensional steady state heat conduction, through a cylindrical wall with internal heat generation in the radial direction, is governed by the cylindrical form of Poisson's equation [6], that is

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+\dot{q}=0. \tag{1}$$

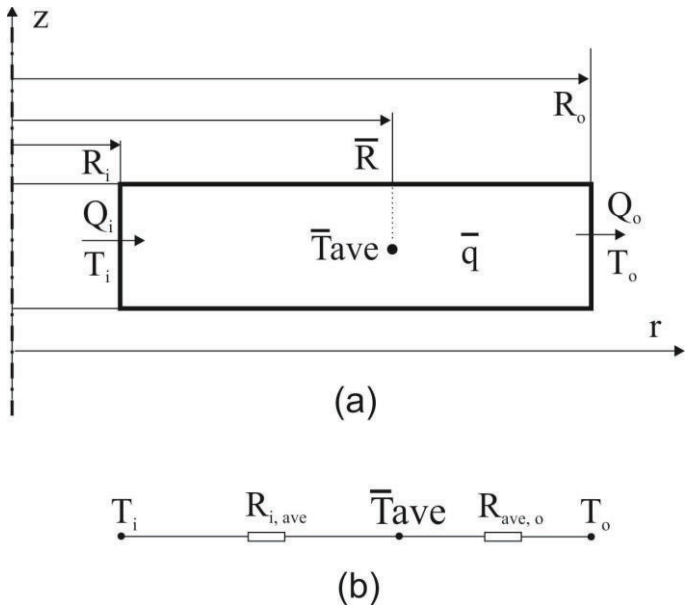

Fig. 1 (a) A Hollow Cylindrical Pellet with a Uniform Nuclear Heat Generation Rate and (b) The Thermal Resistances Network Model.

For the sake of a generality, the inner surface of the hollow cylinder is subjected to a heat transfer rate into the cylinder per unit axial length, Q_i, while the cylinder outer surface temperature is given as T_o. Then the hollow cylinder is subjected to the following thermal boundary conditions,

$$-(2\pi R_i)k\frac{dT}{dr} = Q_i \quad at\ r = R_i; \tag{2}$$

$$T = T_0 \quad at\ r = R_o. \tag{3}$$

Integrated twice, the solution to Eq.1, taking into account the boundary conditions of (2) and (3), is of the form

$$T(r) = T_o + \frac{\dot{q}}{4k}\left({R_o}^2 - r^2\right) + \frac{\dot{q}R_i^2}{2k} ln\left(\frac{r}{R_o}\right) - \frac{Q_i}{2\pi k} ln\left(\frac{r}{R_o}\right). \tag{4}$$

The temperature distribution can also be expressed by the inner surface temperature of the hollow cylindrical pellet, T_i, provided it is known, that is

$$T(r) = T_i - \frac{\dot{q}}{4k}\left(r^2 - {R_i}^2\right) + \frac{\dot{q}R_i^2}{2k} ln\left(\frac{r}{R_i}\right) - \frac{Q_i}{2\pi k} ln\left(\frac{r}{R_i}\right). \tag{5}$$

From equation (4) or (5) it is of importance to recognize that, for one-dimensional, steady state heat conduction in a hollow cylindrical pellet, with an uniform volumetric internal nuclear heat generation rate of $\dot{q}$, a constant thermal conductivity of k and a constant heat transfer rate per unit axial length at the inner surface of the hollow cylindrical pellet, Q_i, the temperature profile within the cylindrical pellet can be determined if and only if the temperature at any cylindrical surface is known.

The above solution is developed for a prescribed heat transfer rate per unit axial length at the inner surface of the hollow cylinder as shown in Eq. (2). For an adiabatic boundary condition, we can just set the heat transfer rate at the prescribed surface to equal to zero.

The above solution is also a general result for a hollow cylinder with an inner radius of R_i. For a solid cylinder, the inner radius can be simply set to zero, saying R_i=0 for Eq. (4). Under this condition, the heat transfer rate per unit axial length at the inner surface of the hollow cylinder becomes also to zero for the reason of symmetry.

Now we have the temperature profile, if we consider the appropriate form of Fourier's law, under steady condition, the rate at which heat is conducted across any cylindrical surface per unit axial length may be expressed as

$$Q(r) = -kA\frac{dT}{dr} = \dot{q}\cdot\pi\left(r^2 - R_i^2\right) + Q_i. \tag{6}$$

2.2 Mean Temperature

General the mean temperature should be calculated by

$$\bar{T}_{ave} = \frac{1}{(\bar{\rho}\bar{c}V)}\int_0^V [\rho c T(r)]dV. \tag{7}$$

If the density and heat capacity are constant, the mean temperature of the above hollow cylinder can be calculated using

$$\bar{T}_{ave} = \frac{1}{V}\int_0^V T(r)\,dV. \tag{8}$$

Hence the mean temperature of the hollow cylinder can be evaluated to give

$$\bar{T}_{ave} = T_o + \frac{\dot{q}}{8k}\left(R_o^2 - R_i^2\right) - \frac{\dot{q}R_i^2}{4k}\left[1 - 2\frac{R_i^2}{\left(R_o^2 - R_i^2\right)}ln\left(\frac{R_o}{R_i}\right)\right] + \frac{Q_i}{4\pi k}\left[1 - 2\frac{R_i^2}{\left(R_o^2 - R_i^2\right)}ln\left(\frac{R_o}{R_i}\right)\right]. \tag{9}$$

As an alternative, the mean temperature can be expressed by the inner surface temperature, T_i, of the hollow cylinder as well, that is

$$\bar{T}_{ave} = T_i - \frac{\dot{q}}{8k}\left(R_o^2 - {R_i}^2\right) - \frac{\dot{q}R_i^2}{4k}\left[1 - 2\frac{R_i^2}{\left(R_o^2 - R_i^2\right)}ln\left(\frac{R_o}{R_i}\right)\right] + \frac{Q_i}{4\pi k}\left[1 - 2\frac{R_i^2}{\left(R_o^2 - R_i^2\right)}ln\left(\frac{R_o}{R_i}\right)\right]. \tag{10}$$

From the results of equations of (9) and (10) it can be known, that the mean temperature of a hollow cylinder can be expressed by its inner or outer surface temperature, if the inner and outer radii of the hollow cylinder are predetermined.

As augmented for the temperature profile, the expression for mean temperature can be also applied to an adiabatic boundary condition at inner surface, and also for solid cylinder as well, if the heat transfer rate at inner surface equals to zero, or the inner radius is set to zero, respectively.

2.3 Mean Temperature Location

The location of the mean temperature position of the hollow cylinder, $\bar{R}$, can be determined by a comparison of temperatures of $T(r) = \bar{T}_{ave}$, that is given by comparison of equations (4) and (9), or comparison of (5) and (10),

$$\bar{R}^2 - 2\left[R_i^2 - \frac{Q_i}{\pi \cdot \dot{q}}\right] ln\left(\frac{\bar{R}}{R_o}\right) - \left\{\frac{1}{2}\left(R_o^2 + R_i^2\right) + \left(R_i^2 - \frac{Q_i}{\pi \cdot \dot{q}}\right)\left[1 - 2\frac{R_i^2}{\left(R_o^2 - R_i^2\right)} ln\left(\frac{R_o}{R_i}\right)\right]\right\} = 0. \quad (11)$$

The location of the mean temperature position ($\bar{R}$) of the hollow cylinder is situated at the radius of (11). It can be proven that if Q_i and $\dot{q}$ are greater than zero, there is only one solution. It should be iteratively calculated, such as with Newton's iteration method.

Unlike the argumentation for temperature distribution and mean temperature expression, eq. (11) cannot be applied to determine the location of the mean temperature position for solid cylinder, namely if the inner radius is vanished. In this case, the location of the mean temperature position can be analytically solved, that is

$$\bar{R} = \frac{1}{\sqrt{2}} R_0. \quad (12)$$

From equation (11) and (12) it is evident that the location of the mean temperature position of the hollow cylinder or solid cylinder is a pure geometrical parameter, provided the heat flux at inner surface and volumetric heat generation rate are known for hollow cylinder. It is independent on material thermal properties. It is of advantage for the computer code since it isn't needed to dynamically calculate the nodal position of the mean temperature for quasi-steady calculation.

2.4 Re-Production of The Temperature Profile

As mentioned above, the temperature profile in the hollow cylinder can be determined if and only if the temperature at any cylindrical surface within the component is known. It is of great advantage if the mean temperature and the mean temperature position are known. With this argumentation we can re-produce the temperature distribution inside the cylinder. Referring to equation (4) or (5), the temperature profile is given

$$T(r) = \bar{T}_{ave} + \frac{\dot{q}}{4k}\left(\bar{R}^2 - r^2\right) + \frac{\dot{q}R_i^2}{2k} ln\left(\frac{r}{\bar{R}}\right) - \frac{Q_i}{2\pi k} ln\left(\frac{r}{\bar{R}}\right). \quad (13)$$

For solid cylinder, the temperature profile can also be re-produced by Eq. (13), if the inner radius, R_i, is set to zero and the location of the mean temperature position is determined by Eq. (12). As long as Q_i is set to zero, the result corresponds to a thermal adiabatic boundary condition at the inner surface.

2.5 Heat Transfer Coefficient and Thermal Resistances

In the lumped parameter thermal circuit method, the thermal-physical properties, temperatures etc. of each component or node are considered to be concentrated at a nodal point within this component. In contrast to the convenient concentrated method [7], in this new model the thermal resistance for each component is concentrated into two parts as shown in Fig.1 (b): between inner surface of hollow cylinder and mean temperature location a thermal resistance element is assigned, while between mean temperature location and outer surface of the hollow cylinder another thermal resistance is prescribed.

Concerning to Fig.1 (b), at first, we consider the heat transfer rate between inner surface and mean temperature node, the representative temperatures are T_i and $\bar{T}_{ave}$, which are suited at radii of R_i and $\bar{R}$, respectively.

Remembering that the thermal resistance is defined as the temperature difference between two nodes divided by the heat transfer rate in-betweens, we obtain

$$\frac{1}{h_{i,ave}(2\pi R_i)} = R_{i,ave} = \frac{T_i - \bar{T}_{ave}}{Q_i}. \quad (14)$$

In this model the heat transfer rate is assigned to sit at radius of $r = R_i$. Next we consider the thermal resistance between mean temperature node and outer surface, accordingly, the representative temperatures are $\bar{T}_{ave}$ and T_o respectively, it gives

$$\frac{1}{h_{ave,o}(2\pi R_o)} = R_{ave,o} = \frac{\bar{T}_{ave} - T_o}{Q_o}. \quad (15)$$

Here the heat transfer rate for this thermal resistance is assigned to sit at radius of $r = R_o$.

3. LUMPED THERMAL RESISTANCE MODEL FOR FUEL PIN STRUCTURE

The same physical description and thermal resistance concept are applied for fuel pin heat transfer model by using an equivalent thermal network of lumped parameter method.

For the sake of simplify the thermal radiations and convective heat transfers in gap are neglected. A steady state is already established, saying at time=0. In the following all the expressions are assumed at time=0 and a unit axial length is considered. We calculate the respective thermal resistance in the following.

3.1 Fuel Pin Pellet

Referring to Fig.2, the fuel pellet forms a hollow cylinder with internal heat generation. It is assumed the inner and outer radii of fuel pellet equal to R_{pi} and R_{po} respectively. A uniform internal heat generation rate per unit volume inside fuel pellet, $\dot{q}_p$, is considered, thus the linear power in the fuel pellet is $Q_p = \dot{q}_p \cdot \pi\left(R_{po}^2 - R_{pi}^2\right)$. Furthermore, at inner surface of pellet of $r = R_{pi}$ it is thermal insulated, a constant temperature, T_{po}, at the outer surface of pellet at $r = R_{po}$ is applied. The pellet mean temperature is assigned to $\bar{T}_p$, which locates at radius of $\bar{R}_f$. With this postulation and Eqs. (9) and (10), the fuel pellet mean temperature is given

$$\bar{T}_p - T_{po} = \frac{\dot{q}_p}{8k_p}\left(R_{po}^2 - R_{pi}^2\right) - \frac{\dot{q}_p \cdot R_{pi}^2}{4k_p} \cdot B_{pi}, \quad (14)$$

Or

$$T_{pi} - \bar{T}_p = \frac{\dot{q}_p}{8k_p}\left(R_{po}^2 - R_{pi}^2\right) + \frac{\dot{q}_p \cdot R_{pi}^2}{4k_p} \cdot B_{po}. \quad (15)$$

Where

$$B_{pi} = 1 - 2\frac{R_{pi}^2}{\left(R_{po}^2 - R_{pi}^2\right)} ln\left(\frac{R_{po}}{R_{pi}}\right), \quad (16)$$

and

$$B_{po} = 1 - 2\frac{R_{po}^2}{\left(R_{po}^2 - R_{pi}^2\right)} ln\left(\frac{R_{po}}{R_{pi}}\right). \quad (17)$$

Two lumped thermal resistances $R_{pi,f}$ and $R_{f,po}$, are introduced to account for the heat transfer between T_{pi} and $\bar{T}_p$, and between $\bar{T}_p$ and T_{po}, respectively.

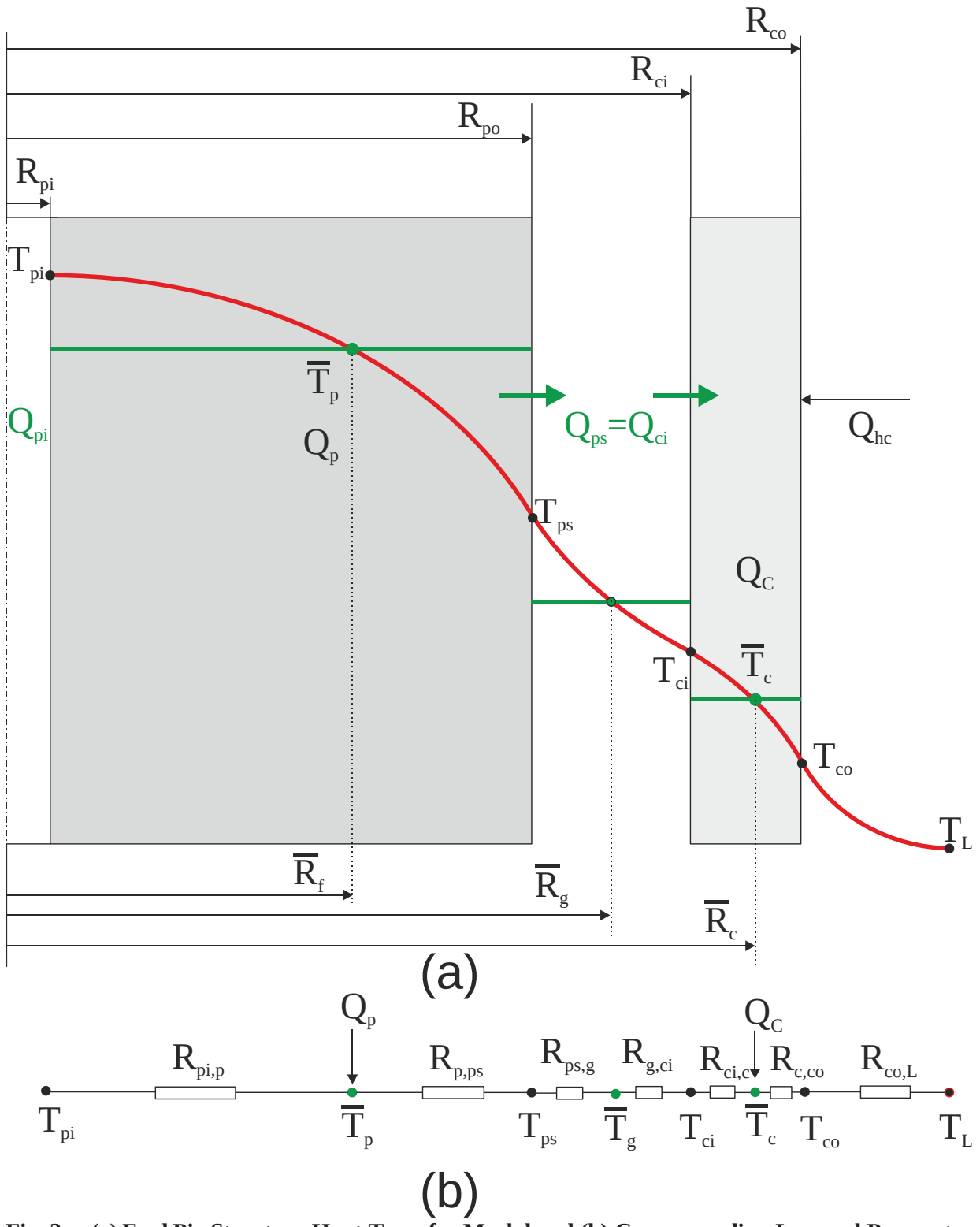

Fig. 2. (a) Fuel Pin Structure Heat Transfer Model and (b) Corresponding Lumped Parameter Thermal Network.

Fig. 2 shows the one dimensional fuel pin structure heat transfer model and its corresponding lumped parameter thermal circuits.

We concentrate now on the energy balance for pin pellet. The heat transfer rate per unit length from inner hole to pellet is assumed to zero, $Q_{pi} = 0$, at $r = R_{pi}$, while the one is denoted as $Q_{po} = \dot{q}_p \cdot \pi\left(R_{po}^2 - R_{pi}^2\right)$ at $r = R_{po}$ from pin pellet to gap. Accordingly, the corresponding heat transfer areas per unit length are $A_{pi} = 2\pi R_{pi}$ and $A_{po} = 2\pi R_{po}$, respectively. With these postulations, the thermal resistances, $R_{pi,f}$ and $R_{f,po}$ are given as follows:

$$\frac{1}{h_{pi,p}\left(2\pi R_{pi}\right)} = R_{pi,p} = \infty, \tag{18}$$

$$\frac{1}{h_{p,po}\left(2\pi R_{po}\right)} = R_{p,po} = \frac{1}{8\pi k_p} - \frac{R_{pi}^2}{4\pi k_p\left(R_{po}^2 - R_{pi}^2\right)} \cdot B_{pi}. \tag{19}$$

3.2 Gap Between Pellet and Cladding

Between pellet and cladding forms a hollow cylinder with inner and outer radii of R_{po} and R_{ci} respectively, as shown in Fig.2. It is assumed that the gap is filled with fission gas (or noble gas) and with a constant thermal conductivity of k_g, furthermore no heat is generated in gap. Under this assumption, the linear heat conduction rate inside gap keeps constant in the radial direction, namely $Q_{po} = Q_{ci}$.

Under this circumstance, one dimensional steady state heat conduction is governed by the cylindrical form of Laplace's equation [7] in the radial direction and the temperature profile is

$$T(r) = T_{ci} - \frac{Q_{ci}}{2\pi k_g} ln\left(\frac{r}{R_{ci}}\right), \tag{20}$$

or

$$T(r) = \bar{T}_g - \frac{Q_{ci}}{2\pi k_g} ln\left(\frac{r}{\bar{R}_g}\right). \tag{21}$$

Though the gap is small compared with pellet size, but the gas thermal conductivity diminishes rapidly with burnup because of the fission product gases Xe & Kr are considerably less than that of the fission gas (normally helium), [8, 9]. In this case two lumped thermal resistances, $R_{po,g}$, and $R_{g,ci}$, should be introduced to account for the heat transfer between pellet outer surface and gap, and between gap and cladding inner surface. With some mathematical manipulation, that is easy to show

$$\frac{1}{h_{po,g}\left(2\pi R_{po}\right)} = R_{po,g} = \frac{T_{po} - \bar{T}_g}{Q_{po}} = \frac{1}{4\pi k_g}\left\{2ln\left(\frac{R_{ci}}{R_{po}}\right)\left[1 + \frac{R_{po}^2}{\left(R_{ci}^2 - R_{po}^2\right)}\right] - 1\right\}, \tag{22}$$

$$\frac{1}{h_{g,ci}\left(2\pi R_{ci}\right)} = R_{g,ci} = \frac{\bar{T}_g - T_{po}}{Q_{ci}} = \frac{1}{4\pi k_g}\left[1 - \frac{2R_{po}^2}{\left(R_{ci}^2 - R_{po}^2\right)} ln\left(\frac{R_{ci}}{R_{po}}\right)\right]. \tag{23}$$

Accordingly, the gap mean temperature and its location are given

$$\bar{T}_g = T_{ci} + \frac{Q_p}{4\pi k_g}\left[1 - \frac{2R_{po}^2}{\left(R_{ci}^2 - R_{po}^2\right)} ln\left(\frac{R_{ci}}{R_{po}}\right)\right], \tag{24}$$

$$2ln\left(\frac{r}{R_{ci}}\right) - \frac{2R_{po}^2}{\left(R_{ci}^2 - R_{po}^2\right)} ln\left(\frac{R_{ci}}{R_{po}}\right) + 1 = 0. \tag{25}$$

3.3 Cladding

The cladding inner and outer radii equal to R_{ci} and R_{co} respectively. A uniform volumetric heat generation rate inside cladding, $\dot{q}_c$, is assumed, thus the linear heat power inside cladding is $Q_c = \dot{q}_c \cdot \pi\left(R_{co}^2 - R_{ci}^2\right)$. At the inner surface of cladding of r= R_{ci} the linear heat conduction rate equals to $Q_{ci} = \dot{q}_p \cdot \pi\left(R_{po}^2 - R_{pi}^2\right)$, a boundary condition of constant temperature, T_{co}, on the outer surface of cladding at r= R_{co} is assumed. A constant thermal conductivity of cladding, k_c, is applied. The cladding mean temperature is assigned as $\bar{T}_c$, which locates at radius of $\bar{R}_c$, as shown in Fig. 2. With these postulations and Eqs. (8) and (9), the cladding mean temperature is given

$$\bar{T}_c - T_{co} = \frac{\dot{q}_c}{8k_c}\left(R_{co}^2 - R_{ci}^2\right) - \frac{\dot{q}_c R_{ci}^2}{4k_c} \cdot B_{ci} + \frac{Q_{ci}}{4\pi k_c} \cdot B_{ci}, \tag{26}$$

Or

$$T_{ci} - \bar{T}_c = \frac{\dot{q}_c}{8k_c}\left(R_{co}^2 - R_{ci}^2\right) + \frac{\dot{q}_c R_{ci}^2}{4k_c} \cdot B_{co} - \frac{Q_{ci}}{4\pi k_c} \cdot B_{co}. \tag{27}$$

Where

$$B_{ci} = 1 - 2\frac{R_{ci}^2}{\left(R_{co}^2 - R_{ci}^2\right)} ln\left(\frac{R_{co}}{R_{ci}}\right); \tag{28}$$

$$B_{co} = 1 - 2\frac{R_o^2}{\left(R_{co}^2 - R_{ci}^2\right)} ln\left(\frac{R_{co}}{R_{ci}}\right). \tag{29}$$

For cladding two lumped thermal resistances, $R_{ci,c}$ and $R_{c,co}$, are introduced to account for the heat transfer between T_{ci} and $\bar{T}_c$, and between $\bar{T}_c$ and T_{co}, respectively, referring to the relations of $Q_p = \dot{q}_p \cdot \pi\left(R_{po}^2 - R_{pi}^2\right)$ and $Q_c = \dot{q}_c \cdot \pi\left(R_{co}^2 - R_{ci}^2\right)$, the thermal resistances are given

$$\frac{1}{h_{ci,c}(2\pi R_{ci})} = R_{ci,c} = \frac{T_{ci} - \bar{T}_c}{Q_{ci}} = \frac{1}{4\pi k_c}\left[\frac{Q_c}{2Q_p} - \left(1 - \frac{\dot{q}_c \pi R_{ci}^2}{Q_f}\right) \cdot B_{co}\right], \tag{30}$$

$$\frac{1}{h_{c,co}(2\pi R_{co})} = R_{c,co} = \frac{\bar{T}_c - T_{co}}{Q_p + Q_c} = \frac{1}{4\pi k_c}\left\{\frac{Q_c}{2(Q_p + Q_c)} + \left(1 - \frac{\dot{q}_c \pi R_{co}^2}{Q_p + Q_c}\right) \cdot B_{ci}\right\}. \tag{31}$$

3.4 Cladding-Fluid Thermal Resistance

The cladding outer surface is cooled by the surrounding fluid, with an average temperature of $\bar{T}_L$. It is assumed that the convective heat transfer coefficient between fluid and cladding outer surface is h_L, at steady state, the heat generated in the fuel pellet and cladding are transferred to the coolant. The heat transfer in-between is defined by the expression of Newton's law of cooling for unit axial length:

$$Q_p + Q_c = h_L(2\pi R_{co})(T_{co} - \bar{T}_L). \tag{32}$$

Where the heat transfer area per unit length is assigned at $r = R_{co}$, accordingly, the thermal resistance is

$$R_{co,L} = \frac{1}{h_L(2\pi R_{co})}. \tag{33}$$

4. LUMPED THERMAL RESISTANCE HEAT TRANSFER EQUATIONS

Suppose the fuel pin is axially divided into "N" nodes, at a given axial pin node. It is assumed that only per unit axial length of mass of different materials are considered, based on the energy balance for individual component, lumped heat transfer equations for fuel pellet, gap and cladding are,

$$\left(m'_p c_p\right)\frac{d\bar{T}_p(t)}{dt} = -\frac{\bar{T}_p(t)-\bar{T}_g(t)}{\left(R_{p,po}+R_{po,g}\right)} + Q_p, \tag{34}$$

$$\left(m'_g c_g\right)\frac{d\bar{T}_g(t)}{dt} = \frac{\bar{T}_p(t)-\bar{T}_g(t)}{\left(R_{p,po}+R_{po,g}\right)} - \frac{\bar{T}_g(t)-\bar{T}_c(t)}{\left(R_{g,ci}+R_{ci,c}\right)}, \tag{35}$$

$$\left(m'_c c_c\right)\frac{d\bar{T}_c(t)}{dt} = \frac{\bar{T}_g(t)-\bar{T}_c(t)}{\left(R_{g,ci}+R_{ci,c}\right)} - \frac{\bar{T}_c(t)-\bar{T}_L(t)}{R_{c,co}+R_{co,L}} + Q_c. \tag{36}$$

where $\left(m'_p c_p\right)$, $\left(m'_g c_g\right)$ and $\left(m'_g c_c\right)$ are the thermal capacities per unit axial length for fuel pellet, gap and cladding, respectively.

For a given axial node the primary coolant energy conservative equation can be written, analog to the [10],

$$\left(m'_L c_L\right)\frac{d\bar{T}_L(t)}{dt} = \frac{\bar{T}_c(t)-\bar{T}_L(t)}{R_{c,co}+R_{co,L}} - 2\dot{m}_L c_L\left(\bar{T}_L(t) - T_{Li}\right). \tag{37}$$

Where $\bar{T}_L(t)$ is tha average coolant temperature at the given axial node, T_{Li} is coolant inlet temperature into the given axial node. $\dot{m}_L$ represents the coolant mass flow rate per unit length. $m'_L \; and \; c_L$ are the coolant mass per unit axial length and heat capacity, respectively.

Proved an initial equilibrium steady state is established at t=0, Eqs. (34) to (36) form a quasi-static lumped thermal resistance transient heat transfer model for fuel pin structure without any other approximations.

If the surrounding fluid average temperature, $\bar{T}_L(t)$, and the correlation of the Newton's law of cooling, namely the thermal resistance $R_{co,L}$, are previously given, Eqs. (34) to (36) form a system of four linear algebraic equations with four unknowns, $\bar{T}_p(t), \bar{T}_g(t), \bar{T}_c(t)$, and $\bar{T}_L(t)$ with the initial conditions of $\bar{T}_p(0)$, $\bar{T}_g(0)$, $\bar{T}_c(0)$ and $\bar{T}_L(0)$. When $\bar{T}_p(0)$, $\bar{T}_g(0)$ and $\bar{T}_c(0)$ are solved, with the help of Eqs. (10), (11) and (12) the temperature profiles inside pin structure can be given.

5. VERIFICATION AND VALIDATION

5.1 Steady State Results

Proved at t=0 a thermal equilibrium state is established, let the left hand side of the Eqs. (34) to (36) equal to zero, we have the steady state results, which form a system of three linear algebraic equations with three unknowns, $\bar{T}_p(0)$, $\bar{T}_g(0)$, and $\bar{T}_c(0)$,

$$\bar{T}_p(0) - \bar{T}_g(0) = Q_p R_{p,g}, \tag{38}$$

$$R_{g,c}\bar{T}_p(0) - \left(R_{p,g} + R_{g,c}\right)\bar{T}_g(0) + R_{p,g}\bar{T}_c(0) = 0, \tag{39}$$

$$-R_{c,L}\bar{T}_g(0) + \left(R_{g,c} + R_{c,L}\right)\bar{T}_c(0) = Q_c\left(R_{g,c}R_{c,L}\right) + R_{g,c}\bar{T}_L(0) \quad (40)$$

Where $R_{p,g} = R_{p,po} + R_{po,g}$; $R_{g,c} = R_{g,ci} + R_{ci,c}$, and $R_{c,L} = R_{c,co} + R_{co,L}$.

The typical pin structure geometry of the advanced sodium technological reactor for industrial demonstration (ASTRID like) [11] developed in Europe is selected to illustrate the application of this model. The main pin structure geometry and thermal properties are listed in table I. Steady state heat transfer between sodium and cladding with the following coolant Newton's law of cooling [12]:

$$Nu = 0.025Re^{0.8}Pr^{0.8}. \quad (41)$$

The average coolant Reynolds number is calculated to be $6.7127\text{x}10^4$ and Prandt number is selected as $4.30\text{x}10^{-3}$, [13]. The gap is filled with helium [8] and the thermal properties used the values from [13].

Table I. Pin structure geometry and thermal properties

Description	Values	Description	Values
Inner hole diameter at 20°C (mm)	2.2	Outer pellet diameter at 20°C (mm)	8.45
Cladding inner diameter at 20°C (mm)	8.7	Cladding outer diameter at 20°C (mm)	9.7
Pellet linear power (W/m)	3.6086×10^4	Volumetric Power of pellet (W/m^3)	6.9027×10^8
Cladding linear power (w/m)	9.0215×10^1	Volumetric Power of cladding (W/m^3)	6.2427×10^6
Sodium heat transfer coefficient ($W/m^2\cdot K$)	4.3883×10^4	Sodium average temperature (°C)	500
Pellet thermal conductivity (W/m·k)	2.31	Gap thermal conductivity (W/m·k)	0.3762
Cladding thermal conductivity (W/m·k)	17.1	Sodium thermal conductivity (W/m·k)	69
Pellet density (kg/m^3)	10641	Pellet thermal capacity (J/kg·k)	334
Cladding density (kg/m^3)	7864	Cladding thermal capacity (J/kg·k)	529
Gap (helium) density (kg/m^3)	0.04486	Gap thermal capacity(J/kg·k)	5193
Sodium density (kg/m^3)	832	Sodium thermal capacity (J/kg·k)	529
Sodium mass flow rate (kg/s)	0.2863		

Table II. Different thermal resistance

Description	Thermal resistance	Symbol	Values
Pellet	Inner surface to mean temperature location	$R_{pi,p}$	∞
	Mean temperature location to outer surface	$R_{p,po}$	1.5210×10^{-2}
Gap	Pin outer surface to mean temperature	$R_{po,g}$	6.2274×10^{-3}
	Mean temperature to cladding inner surface	$R_{g,ci}$	6.1075×10^{-3}
Cladding	Cladding inner surface to mean temperature	$R_{ci,c}$	5.2510×10^{-4}
	Mean temperature to cladding outer surface	$R_{c,co}$	4.8756×10^{-4}
Sodium	Cladding outer surface to coolant	$R_{co,L}$	7.4779×10^{-4}

Table II gives the calculated different thermal resistance for this pin structure geometry and the calculated mean temperature location and the mean temperature are given in table III and illustrated in Fig. 3.

Table III. Calculated steady state results

Description	°C	Mean temperature location radius (mm)
Pellet mean temperature, $\bar{T}_p(0)$	1557.62	3.1265
Gap mean temperature, $T_g(0)$	784.036	4.2876
Cladding mean temperature, $T_c(0)$	544.69	4.6023

5.2 Comparison With Analytical Solution and Re-Production of The Temperature Profile

As mentioned above, the temperature at any cylindrical surface in the hollow cylinder can be determined if and only if the temperature at one cylindrical surface within the corresponding element is known. We use here the calculation results of the table 3 to re-produce the temperature profile for the pin structure. Referring to equation (13), the temperature profile is calculated and shown in Fig. 3, in which the analytical temperature distribution and the calculated location of the mean temperature positions of the pellet, $\bar{T}_p$, gap, $\bar{T}_g$, and the cladding, $\bar{T}_c$, and their values are also given. It can be seen that the mean temperatures of each part are exactly suited on the analytical temperature profile.

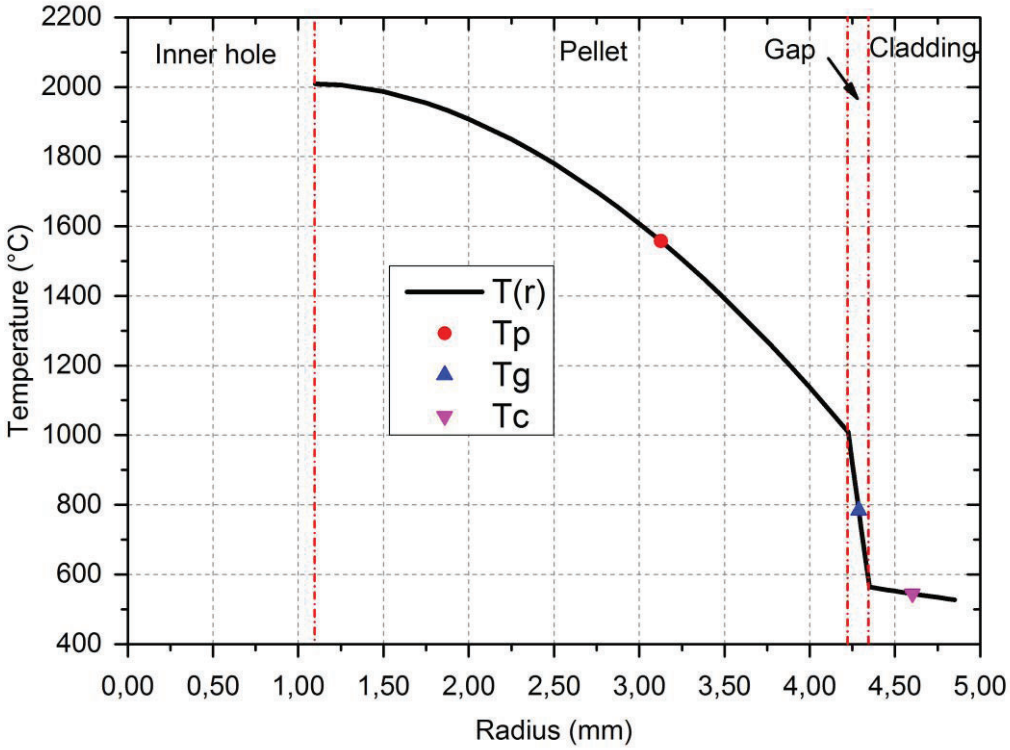

Fig. 3 Analytical Temperature Profile and Average Temperatures for Pellet, Gap, and Cladding.

6. CONCLUSION

A generalized lumped parameter thermal resistance method has been developed for pin structure with uniform volumetric internal heat generation. The treatment has been performed by considering the law of energy conservation between lumped mean temperature node and element surfaces. For the definition of the thermal resistance, the heat conduction between lumped mean temperature node of one component and neighborhoods take place on the corresponding element surfaces, at which the heat transfer areas are defined. Between each element surface and lumped mean temperature node a thermal resistance is assigned, the thermal resistance connects the mean temperature node with the corresponding surface. The mean temperature is purposely and exactly set up on the analytical temperature profile; with this arrangement all the thermal resistances and the location of the mean temperature position can be determined analytically. The advantage of the presented method is that the temperature distributions across whole pin structure can be re-produced after a quite easily lumped heat transfer calculation as illustrated in the section 5.2.

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