THE DYNAMICAL SYSTEM SCALING METHODOLOGY: COMPARING DIMENSIONLESS GOVERNING EQUATIONS WITH THE H2TS AND FSA METHODOLOGIES

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Abstract

The Dynamical System Scaling (DSS) methodology is an innovative scaling approach that, as the name implies, incorporates the system's dynamic response into the scaling framework. The DSS methodology has an important benefit in that scale distortion is a dynamic quantity and can therefore change through time. Depending on the system and transient of interest, scale distortion may decrease through time allowing engineers to recover test facility data that would have been previously considered ill-scaled. The lead author has a separate paper in this Topical Meeting that introduces the DSS formalism and presents the DSS distortion measure. The goal of this paper is to explicitly contrast the DSS dimensionless governing equations to the dimensionless governing equations corresponding to the Hierarchical Two-Tiered Scaling (H2TS) and Fractional Scaling Analysis (FSA) methodologies and ultimately compare the DSS distortion measure to the H2TS and FSA distortion measures. The resulting similarity criteria are compared to show that the DSS similarity criteria includes the dynamic response rather than only using reference values established at initial, average or final conditions. After providing comparisons with generalized balance equations, the scaling methods are compared for a single-phase natural circulation scenario. Depending on the transient, the H2TS and FSA similarity criteria may depend on engineering judgment for how the reference values were chosen. DSS completely bypasses that issue as noted in the discussions presented within this paper. This paper compares the DSS methodology for a single dynamical process. This paper is one among a series that will compare and contrast the DSS methodology with the H2TS and FSA methods for a variety of practical engineering applications including the application of the DSS methodology to a hierarchical systems with coupled components.

Keywords: Scaling analysis, dynamical systems, time-dependent scale distortion, H2TS, FSA

1. INTRODUCTION

The Hierarchical Two-Tiered Scaling (H2TS) methodology and more recently the Fractional Scaling Analysis (FSA) methodology are well established techniques that have been adopted in many studies [1]-[7]. To assess time-dependent scaling effects, recent studies [8]-[9] have extended these methods to include dimensionless groups based on the initial conditions, average conditions, or final conditions. A method for assessing time dependent scaling distortions evaluated as the transient evolves is discussed in [10]. The Dynamical System Scaling (DSS) approach presented in this paper is a powerful new methodology that builds from the H2TS and FSA approaches by incorporating the system's dynamic response into the scaling framework. The DSS formalism, however, may at first appear difficult or unfamiliar to the thermal-hydraulic community since it is based on concepts derived from differential geometry which are more commonly applied in general relativity or other branches of science.

The purpose of this paper is to outline the strong parallel and consistency among the DSS, the H2TS and FSA methods. Ultimately DSS provides a more complete analysis by folding the dynamic aspect in the development of similarity criteria.

2. GOVERNING EQUATIONS

2.1 Theory

The integral balance equation is in a form consistent with the work by Reyes [11]:

$$\frac{d}{dt}\left\{\iiint \psi\left(\underline{x},t\right)dV\right\} = \sum_{i=1}^{n}\varphi_i \tag{1}$$

Where $\psi\left(\underline{x},t\right)$ is the local instantaneous amount of the conserved quantity, such as density in the continuity equation, and φ_i is the *i*-agent-of-change acting to change the conserved quantity. The space-integrated conserved quantity $\Psi\left(t\right)$ is the instantaneous amount of the conserved quantity at a particular point in time, and is defined as:

$$\Psi\left(t\right) = \iiint \psi\left(\underline{x},t\right)dV \tag{2}$$

In DSS, the normalized conserved quantity is defined as:

$$\beta\left(t\right) = \frac{\Psi\left(t\right)}{\Psi_0} = \frac{\iiint \psi\left(\underline{x},t\right)dV}{\Psi_0} \tag{3}$$

The normalized sum of agents-of-change is defined as:

$$\omega\left(t\right) = \frac{1}{\Psi_0}\sum_{i=1}^{n}\varphi_i = \sum_{i=1}^{n}\frac{\varphi_i}{\Psi_0} \tag{4}$$

The DSS normalized integral balance equation is simply:

$$\frac{d\beta}{dt} = \omega \tag{5}$$

This is contrasted with the FSA approach, which starts with the same integral balance equation:

$$\frac{d\Psi}{dt} = \sum_{i=1}^{n} \varphi_i \tag{6}$$

The quantity Ψ is scaled by some reference value Ψ_0 as:

$$\Psi^+ = \frac{\Psi}{\Psi_0} \tag{7}$$

Leading to the normalized integral balance equation:

$$\frac{d\Psi^+}{dt} = \frac{1}{\Psi_0} \sum_{i=1}^{n} \varphi_i \tag{8}$$

Note that as long as Ψ_0 is the same reference value used in both FSA and DSS, then up to this point DSS and FSA are the same:

$$\frac{d\beta}{dt} = \frac{d\Psi^+}{dt} \tag{9}$$

The DSS normalized sum of agents-of-change is equivalent to the R.H.S. of Eq. (8), as previously shown by Eq. (4):

$$\omega = \frac{1}{\Psi_0} \sum_{i=1}^{n} \varphi_i \tag{10}$$

Zuber [1] describes two approaches to scale the agent-of-change:

a) The standard approach which is equivalent to the H2TS
b) The aggregate approach using the effective Fractional Rate of Change (FRC)

In the standard approach (H2TS) the individual agents-of-change are scaled by their own, individual, reference values:

$$\varphi_i^+ = \frac{\varphi_i}{\varphi_{i,0}} \tag{11}$$

In the FSA approach the effective agent-of-change is defined as the sum of all agents-of-change:

$$\varphi_e = \sum_{i=1}^{n} \varphi_i \tag{12}$$

This is related to the DSS sum of agents-of-change by:

$$\omega = \frac{\varphi_e}{\Psi_0} \tag{13}$$

However the FSA approach scales each agent-of-change by the effective (sum of) agents of change, defined at specific reference values for the individual agents-of-change.

The reference effective agent-of-change is then:

$$\varphi_{e,0} = \sum_{i=1}^{n} \varphi_{i,0} \tag{14}$$

Thus, $\varphi_{e,0}$ is simply the sum of individual reference values used in approach 1 (H2TS). The FSA scaled agents of change are:

$$\varphi_i^* = \frac{\varphi_i}{\varphi_{\mathrm{e},0}} \tag{15}$$

Note that the denominator is often defined as the absolute value [2], i.e.:

$$\varphi_i^* = \frac{\varphi_i}{\left|\varphi_{\mathrm{e},0}\right|} \tag{16}$$

The scaled effective agent-of-change is then:

$$\varphi_e^* = \sum_{i=1}^{n} \varphi_i^* = \frac{1}{\left|\varphi_{\mathrm{e},0}\right|} \sum_{i=1}^{n} \varphi_i = \sum_{i=1}^{n} \frac{\varphi_i}{\left|\varphi_{\mathrm{e},0}\right|} \tag{17}$$

The scaled integral balance equations for approach 1 (H2TS) and approach 2 (FSA) are compared in Table 1.

Table 1 – Comparison between the H2TS and FSA formalism

H2TS	FSA
$\frac{d\Psi^+}{dt} = \frac{1}{\Psi_0} \sum_{i=1}^{n} \varphi_{i,0} \varphi_i^+$	$\frac{d\Psi^+}{dt} = \frac{1}{\Psi_0} \left(\left|\varphi_{\mathrm{e},0}\right| \sum_{i=1}^{n} \varphi_i^* \right)$

From Table 1 and Eq. (9), the relationship between the DSS normalized sum of agents-of-change, and the H2TS and FSA agents-of-change is given by:

$$\omega = \frac{1}{\Psi_0}\sum_{i=1}^{n}\varphi_{i,0}\varphi_i^+ = \frac{\left|\varphi_{\mathrm{e},0}\right|}{\Psi_0}\sum_{i=1}^{n}\varphi_i^* \tag{18}$$

In the following, the reader needs to be aware of a notation change. In FSA, the Fractional Rate of Change (FRC) is defined for each agent-of-change as well as the effective agent-of-change. In FSA the FRC is typically also denoted as ω, so to avoid confusion we will denote the FRC for the *i*-th agent-of-change as $\omega_{FRC,i}$ in the following discussion. More specifically, the FRC for the *i*-th agent-of-change at the reference value is:

$$\omega_{FRC,i,0} = \frac{\varphi_{\mathrm{i},0}}{\Psi_0} \tag{19}$$

This is ratio of the agent-of-change reference value to the conserved quantity reference value.

The effective FRC is simply the sum of the individual FRCs:

$$\omega_{FRC,\mathrm{e},0} = \sum_{i=1}^{n}\omega_{FRC,\mathrm{i},0} = \sum_{i=1}^{n}\frac{\varphi_{\mathrm{i},0}}{\Psi_0} = \frac{\varphi_{\mathrm{e},0}}{\Psi_0} \tag{20}$$

Or its absolute value:

$$\left|\omega_{FRC,\mathrm{e},0}\right| = \frac{\left|\varphi_{\mathrm{e},0}\right|}{\Psi_0} \tag{21}$$

In conclusion, combining with the equations in Table 1, the DSS normalized sum of agent-of-change is related to the FRCs as follows:

$$\underset{\text{DSS}}{\omega} = \underset{\text{H2TS}}{\sum_{i=1}^{n}\omega_{FRC,\mathrm{i},0}\varphi_i^+} = \underset{\text{FSA}}{\left|\omega_{FRC,\mathrm{e},0}\right|\sum_{i=1}^{n}\varphi_i^* = \left|\omega_{FRC,\mathrm{e},0}\right|\varphi_e^*} \tag{22}$$

The following scaled integral balance equation is obtained assuming β was chosen such that $\beta = \Psi^+$:

$$\frac{d\beta}{dt} = \frac{d\Psi^+}{dt} = \omega = \sum_{i=1}^{n}\omega_{FRC,\mathrm{i},0}\varphi_i^+ = \left|\omega_{FRC,\mathrm{e},0}\right|\varphi_e^* \tag{23}$$

The term in the equation has Hz as units.

A significant difference between the three formulations (DSS, H2TS and FSA) is the normalization of the reference time. Typically H2TS does not explicitly define the normalized time, but the dimensionless governing equations make use of time-constant parameters associated with the characteristic time of the process considered in the equation.

Here, the H2TS normalized time follows the work by Zuber in his 2007 NED paper [1] where the normalized time is defined from the largest FRC, which corresponds to the dominant agent-of-change:

$$\omega_{FRC,\max} = \max\left(\left\{\omega_{FRC,i,0}\right\}_{i=1}^{n}\right) \tag{24}$$

The H2TS normalized time is then:

$$t^{+} = \omega_{FRC,\max} t \tag{25}$$

As result, the H2TS dimensionless integral balance Equation is:

$$\frac{d\Psi^{+}}{dt^{+}} = \varphi_{i,\max}^{+} + \sum_{i=1}^{n} \frac{\omega_{FRC,\mathrm{i},0}}{\omega_{FRC,\max}} \varphi_{i}^{+} \tag{26}$$

The first term of the R.H.S is the dimensionless dominant agent-of-change whereas the second term of the R.H.S are the dimensionless agent-of-change for all other agents-of-change. Following the H2TS approach the relative importance of the processes involved can be judged by the ratios of the FRCs relative to the max FRC. If a particular ratio is much less than 1, the corresponding agent-of-change (or process) is not important.

In FSA, the dimensionless time is defined as the effect metric:

$$\Omega_{FSA,e} = \left|\omega_{FRC,e}\right| t \tag{27}$$

Zuber denotes the effect metric as either Ω or Ω_e. Here, again, to clarify that this is not the same as the DSS effect parameter, the suffix FSA was added.

The FSA dimensionless integral balance equation is then:

$$\frac{d\Psi^{+}}{d\Omega_{FSA,e}} = \varphi_{e}^{*} \tag{28}$$

The importance ranking in FSA is performed by relating the individual FRC's to the effective FRC. The effect metric, $\Omega_{FSA,e}$ is used to create the similarity criteria since it captures the relationship of all the agents-of-change.

However the effect metric is still a static quantity which depends only on fixed reference values chosen by the analyst and does not account for the evolution and relationship between the different agents-of-change through time.

The main differentiator between the DSS and H2TS/FSA is the consideration of the dynamic aspect. DSS defines the dimensionless (normalized) time by dividing time by the action, τ_S, which is defined as the integral of the temporal displacement rate, D, over the transient:

$$\tau_S = \int_{t_I}^{t_F} (1+D)dt \tag{29}$$

Where t_I and t_F are the initial and final times, respectively. The temporal displacement rate relates the process time interval to the (reference) time interval with process time defined as:

$$\tau = \frac{\beta}{\omega} \tag{30}$$

Therefore the temporal displacement rate is:

$$D = \frac{d\tau - dt}{dt} \Rightarrow D = \frac{-\beta}{\omega^2}\frac{d\omega}{dt} \tag{31}$$

It can be shown that the temporal displacement rate integral is equal to the change in process time over the transient which also corresponds to the action by virtue of Eq. (29):

$$\int_{t_I}^{t_F} (1+D)dt = \tau_F - \tau_I = \tau_S \tag{32}$$

DSS defines the normalized time as follows:

$$\tilde{t} = \frac{t}{\tau_S} \tag{33}$$

Which gives the DSS dimensionless integral balance equation to be:

$$\frac{d\beta}{d\tilde{t}} = \tau_S \omega \tag{34}$$

The quantity $\tau_S \omega$ is defined in DSS as the <u>effect parameter</u> $\tilde{\Omega}$:

$$\tilde{\Omega} = \tau_S \omega \tag{35}$$

The three methods can then be compared in Table 2.

Table 2 – Comparison between the H2TS, FSA and DSS formalism for effect metric.

H2TS	FSA	DSS
$\frac{d\Psi^+}{dt^+} = \varphi^+_{i,\max} + \sum_{i=1}^{n} \frac{\omega_{FRC,i,0}}{\omega_{FRC,\max}} \varphi_i^+$	$\frac{d\Psi^+}{d\Omega_{FSA,e}} = \varphi_e^*$	$\frac{d\beta}{d\tilde{t}} = \tilde{\Omega}$

In Zuber's [1], he first defined the FRC on a general state-variable undergoing a change by a particular agent. In the present notation the general FRC is:

$$\omega_{FRC} = \frac{\varphi}{\Psi} \tag{36}$$

The effective FRC is similarly defined:

$$\omega_{FRC,e} = \frac{\varphi_e}{\Psi} \tag{37}$$

The previously defined effective FRC, $\omega_{FRC,e,0}$ has the subscript '0' to denote it was defined using the reference values for the agents-of-change and conserved quantity. However, a generalized effective FRC can be defined at any point in time, and related to the DSS process time via:

$$\omega_{FRC,e} = \frac{\varphi_e}{\Psi} = \frac{\sum \varphi_i}{\Psi} = \frac{\Psi_0 \omega}{\Psi} = \frac{\omega}{\Psi^+} = \frac{\omega}{\beta} \tag{38}$$

Therefore, the general effective FRC is the inverse of the DSS process time:

$$\omega_{FRC,e} = \frac{1}{\tau} \tag{39}$$

Note that the FSA effect metric, $\Omega_{FSA,e}$, is defined using $\left|\omega_{FRC,e}\right|$ and requires engineering judgement to define proper various agent-of-change reference values. The reference values are often chosen as the initial value of the transient phase analysed. However this is sometimes not obvious because an initial value could be zero which would make the choice not appropriate.

The DSS effect parameter can be related to the FSA FRC, starting from the following relationship:

$$\omega = \frac{\left|\varphi_{e,0}\right|}{\Psi_0} \varphi_e^* = \left|\omega_{FRC,e,0}\right| \varphi_e^* \tag{40}$$

The R.H.S of Eq. (40) is substituted in Eq. (35):

$$\tilde{\Omega} = \tau_S \omega = \tau_S \left|\omega_{FRC,e,0}\right| \varphi_e^* \tag{41}$$

Eq. (41) provides an interesting insight on the DSS. The DSS effect parameter fully accounts for the evolution of the dimensionless agent-of-change as defined in the FSA.

Eq. (41) can also be expressed in term of the FSA effect metrics from Eq. (27):

$$\tilde{\Omega} = \tau_S \left(\frac{\Omega_{FSA,e}}{t} \right) \varphi_e^* \tag{42}$$

Or

$$\tilde{\Omega} = \frac{\Omega_{FSA,e}}{\tilde{t}} \varphi_e^* \tag{43}$$

Unlike FSA, DSS does not classify similarity using the dimensionless integral balance equation. DSS applies differential geometry and geodesic analysis to compare the response on a dimensionless set of space-time coordinates $\left(\beta - \tilde{\Omega} - \tilde{\tau}\right)$ where $\tilde{\tau}$ is the dimensionless process time defined as follows:

$$\tilde{\tau} = \frac{\tau}{\tau_S} \tag{44}$$

From the definition of the action and process time, the dimensionless process time can be expressed in different ways:

$$\tilde{\tau} = \frac{\tau}{\tau_S} = \frac{\tau}{\tau_F - \tau_I} = \frac{\dfrac{\beta}{\omega}}{\left(\dfrac{\beta}{\omega}\right)_F - \left(\dfrac{\beta}{\omega}\right)_I} = \frac{\left(\beta / \beta_I\right) / \left(\omega / \omega_I\right)}{\dfrac{\left(\beta_F / \beta_I\right)}{\left(\omega_F / \omega_I\right)} - 1} \tag{45}$$

Using Eq. (39) the relationship between the dimensionless process time and the effective fraction rate of change $\omega_{FRC,e}$:

$$\tilde{\tau} = \frac{\tau}{\tau_F - \tau_I} = \frac{\omega_{FRC,e,F}\,\omega_{FRC,e,\mathrm{I}}}{\omega_{FRC,e}\left(\omega_{FRC,e,\mathrm{I}} - \omega_{FRC,e,\mathrm{F}}\right)} \tag{46}$$

2.2 Assessment of Scaling Distortions

Scale distortion in H2TS is simply determined by comparing Π-groups between test facilities at different scales. In the present notation the Π-groups are the ratios of the individual agent-of-change FRCs to the dominant maximum FRC:

$$\Pi_i = \frac{\omega_{FRC,i}}{\omega_{FRC,\max}} \tag{47}$$

As stated before, an unimportant agent-of-change will have a rather small Π-value ($\Pi_i << 1.0$) while a significant one will have $\Pi_i \sim 1.0$. Distortion factors for the specific agent of change will be computed as follows:

$$DF_i = \frac{\Pi_{i,P} - \Pi_{i,\mathrm{M}}}{\Pi_{i,P}} = 1 - \frac{\Pi_{i,\mathrm{M}}}{\Pi_{i,P}} = 1 - \Pi_{i,\mathrm{R}} \tag{48}$$

Where the superscript P stands for 'Prototype", M for "Model" and R for "Ratio". In term of FRCs, the distortion factors are:

$$DF_i = 1 - \frac{\left(\dfrac{\omega_{FRC,i}}{\omega_{FRC,\max}}\right)_M}{\left(\dfrac{\omega_{FRC,i}}{\omega_{FRC,\max}}\right)_P} = 1 - \frac{\left(\omega_{FRC,i}\right)_R}{\left(\omega_{FRC,\max}\right)_R} \tag{49}$$

Where:

$$\omega_R = \frac{\omega_M}{\omega_P} \tag{50}$$

Within FSA, distortion is measured by comparing the effective effect metric $\Omega_{FSA,e}$ between the model and the prototype. Zuber [1] does not explicitly provide an expression for the distortion in FSA but using an expression similar to H2TS, the effective distortion within FSA can be obtained by the following relationship:

$$DF_{FSA,e} = 1 - \frac{\left(\Omega_{FSA,e}\right)_M}{\left(\Omega_{FSA,e}\right)_P} = 1 - \left(\Omega_{FSA,e}\right)_R \tag{51}$$

In other words, in FSA, the distortion relative to the individual agent-of-change can be determined by comparing the specific FRC-values in the model relative to the prototype.

The distortions in DSS are judged by the concept of "geodesic separation" [11] between the model the prototype along the dimensionless $\left(\beta - \tilde{\Omega} - \tilde{\tau}\right)$ space-time coordinates. Due to the complexities of computing the separation along a curved space-time, a flat-space separation distance is used to estimate the instantaneous separation at a particular $\tilde{\tau}$-value:

$$\eta_x\left(\tilde{\tau}\right) = \sqrt{1 + \tilde{\tau}^2}\left(\tilde{\Omega}_P - \frac{\tilde{\Omega}_M}{\lambda_A}\right) \tag{52}$$

The total separation, η_T is computed by integrating the above expression from τ_I to τ_F, yielding the total scale distortion during the transient:

$$\eta_T = \frac{1}{\left(\tilde{\tau}_F - \tilde{\tau}_I\right)} \int_{\tilde{\tau}_I}^{\tilde{\tau}_F} \left|\eta_x\left(\tilde{\tau}\right)\right| d\tilde{\tau} \tag{53}$$

Note that from Eq. (41) the effect parameter varies with time, specifically $\tilde{\Omega} = \tau_S \left|\omega_{FRC,e,0}\right| \varphi_e^*$.
Therefore DSS allows distortion to vary as a function of time, as the particular transient evolves:

$$\eta_x\left(\tilde{\tau}\right) = \sqrt{1+\tilde{\tau}^2} \left\{\left|\omega_{FRC,e,0}\right| \tau_S\right\} \left|\left(\varphi_e^*\right)_P - \left(\varphi_e^*\right)_M\right| \tag{54}$$

In Eq. (54) the quantities τ_S and $\left|\omega_{FRC,e,0}\right|$ are constant but φ_e^* is the dimensionless effective agent-of-change at a point in time. Reference [10] used FSA to compute the distortion through time by computing the effective FRC at numerous time instances during a transient. The FSA dimensionless time at each time instance therefore was computed using different normalizing constants. As shown in Eq. (54), the DSS flat-space separation distance uses a constant normalizing factor, the action, over the particular time period of interest. The flat-space separation distance may change through the transient if the (dimensionless) sum of agents of change are also varying through reference time.

2.3 Example: Single-Phase Natural Circulation in a Loop

Let's consider a single-phase natural circulation around a loop. The problem is described in detail in [4] with the following simplifying assumptions:

1. The flow was one-dimensional along the loop axis; therefore, fluid properties were uniform at every cross-section.
2. The Boussinesq approximation was applicable.
3. The fluid was incompressible.
4. The cold leg temperature is constant.
5. Form losses, primarily in the core and steam generator regions, dominate the loop resistance.
6. Acceleration pressure drop is negligible

Following these assumptions, the momentum balance equation for the loop reduces to:

$$\sum\left(\frac{l_i}{a_i}\right) \cdot \frac{d\dot{m}}{dt} = B_T \rho_l g L_{th} \dot{q}_{core} - \frac{\dot{m}^2}{\rho_l a_c^2} \sum_{i=1}^{n} \left[\frac{1}{2}\left(\frac{fl}{d_h} + K_l\right)\left(\frac{a_c}{a_i}\right)^2\right] \tag{55}$$

And the energy balance equation is:

$$C_{vl} M_{sys} \frac{d}{dt}\left(T_M - T_C\right) = \dot{q}_{core} - \dot{q}_{SG} - \dot{q}_{RPV} \tag{56}$$

Let's define the loop time constant as $\tau_{Loop} = \frac{M_{SYS}}{\dot{m}_0}$ and the reference length as $l_{ref} = \frac{M_{SYS}}{\rho_l a_c}$, where M_{SYS} is the total system mass and a_c is the flow area in the core. Also relate the core power to the hot and cold leg temperature difference:

$$\frac{\dot{q}_{core,0}}{\dot{m}_0 C_{pl}} = \left(T_H - T_C\right)_0, \qquad \frac{\dot{q}_{core}}{\dot{m} C_{pl}} = \left(T_H - T_C\right) \tag{57}$$

Now we define the following dimensionless parameters, the Richardson number Π_{Ri}, the flow resistance number Π_{Fl} and the loop reference length number Π_L:

$$\Pi_{Ri} = \frac{B_T g \left(T_H - T_C\right)_0 L_{th}}{u_{c0}^2} \tag{58}$$

$$\Pi_{Fl} = \left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^2 \right] \right)_0 \tag{59}$$

$$\Pi_L = \left(\frac{a_c}{l_{ref}} \right) \sum \left(\frac{l_i}{a_i} \right) \tag{60}$$

After some manipulation of Eq. (55) the following equation is obtained:

$$\Pi_L \tau_{Loop} \frac{d\dot{m}^+}{dt} = \Pi_{Ri} \left(T_H - T_C\right)^+ - \Pi_{Fl} \left(\dot{m}^+\right)^2 \left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^2 \right] \right)^+ \tag{61}$$

Where:

$$\left(T_H - T_C\right)^+ = \frac{\left(T_H - T_C\right)}{\left(T_H - T_C\right)_0}$$

$$\left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^2 \right] \right)^+ = \frac{\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^2 \right]}{\left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^2 \right] \right)_0} \tag{62}$$

Now there are several choices to define the dimensionless time. Following the more recent work by Zuber [1], the dimensionless time should be defined such that the coefficient to the dominant agent of change is 1.0. At steady-state, the buoyancy and frictional loss forces balance. However should the core power be reduced, the frictional losses will become dominant, leading to a reduction in the loop flow rate over time. The dimensionless time will therefore be defined:

$$t^{+} = \frac{\Pi_{Fl}}{\Pi_{L}} \frac{t}{\tau_{Loop}} \tag{63}$$

The dimensionless momentum equation becomes:

$$\frac{d\dot{m}^{+}}{dt^{+}} = \frac{\Pi_{Ri}}{\Pi_{Fl}} \left(T_H - T_C\right)^{+} - \left(\dot{m}^{+}\right)^{2} \left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^{2} \right] \right)^{+} \tag{64}$$

Which is consistent with the H2TS form presented before in Table 2, repeated here for convenience, $\frac{d\Psi^{+}}{dt^{+}} = \varphi_{i,\max}^{+} + \sum_{i=1}^{n} \frac{\omega_{FRC,\mathrm{i},0}}{\omega_{FRC,\max}} \varphi_{i}^{+} = \varphi_{i,\max}^{+} + \sum_{i=1}^{n} \Pi_{i} \varphi_{i}^{+}$, where:

$$\begin{aligned} &\Psi^{+} = \dot{m}^{+} \\ &\varphi_{i,\max}^{+} = -\varphi_{Fl}^{+} = -\left(\dot{m}^{+}\right)^{2} \left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^{2} \right] \right)^{+} \\ &\sum_{i=1}^{N} \Pi_{i} \varphi_{i}^{+} = \frac{\Pi_{Ri}}{\Pi_{Fl}} \left(T_H - T_C\right)^{+} \end{aligned} \tag{65}$$

Where N is capitalized, because N represents the number of Π-groups (one in this case) and n are the segments in which the loop has been divided. A single dimensionless group can be defined as the ratio of the buoyancy to frictional loss forces, the natural circulation number:

$$\Pi_{NC} = \frac{\Pi_{Ri}}{\Pi_{Fl}} \quad \text{and} \quad \varphi_{NC}^{+} = \left(T_H - T_C\right)^{+} \tag{66}$$

The H2TS dimensionless momentum equation can be compactly written as:

$$\frac{d\dot{m}^{+}}{dt^{+}} = \Pi_{NC} \varphi_{NC}^{+} - \varphi_{Fl}^{+} \tag{67}$$

Before, writing the corresponding DSS equation, the individual normalized agents-of-change are defined by:

$$\omega_{Ri} = \frac{\Pi_{Ri}}{\Pi_{L} \tau_{Loop}} \left(T_H - T_C\right)^{+}, \qquad \omega_{Fl} = \frac{\Pi_{Fl}}{\Pi_{L} \tau_{Loop}} \left(\dot{m}^{+}\right)^{2} \left(\sum_{i=1}^{n} \left[\frac{1}{2} \left(\frac{fl}{d_h} + K_l \right) \left(\frac{a_c}{a_i} \right)^{2} \right] \right)^{+} \tag{68}$$

Multiply Eq. (61) by the action, τ_S to yield the DSS dimensionless momentum equation:

$$\tau_S \frac{d\dot{m}^+}{dt} = \tau_S \left(\omega_{Ri} - \omega_{Fl}\right) \rightarrow \frac{d\dot{m}^+}{d\left(t/\tau_S\right)} = \tilde{\Omega}_{Ri} - \tilde{\Omega}_{Fl} \rightarrow \frac{d\dot{m}^+}{d\tilde{t}} = \tilde{\Omega} \quad (69)$$

A relationship between the DSS effect parameter and the H2TS dimensionless momentum equation can be found by rearranging Eq. (67) and Eq. (69) in the following manner:

$$\frac{\Pi_{Fl}}{\Pi_L \tau_{Loop}} \frac{d\dot{m}^+}{dt^+} = \frac{d\dot{m}^+}{dt} = \omega_{Ri} - \omega_{Fl} = \frac{\tilde{\Omega}}{\tau_S} \quad (70)$$

Therefore, the DSS effect parameter can be written in terms of the H2TS dimensionless momentum equation:

$$\tilde{\Omega} = \frac{\Pi_L}{\Pi_{Fl}} \frac{\tau_{Loop}}{\tau_S} \frac{d\dot{m}^+}{dt^+} = \frac{\Pi_L}{\Pi_{Fl}} \frac{\tau_{Loop}}{\tau_S} \left(\Pi_{NC}\varphi_{NC}^+ - \varphi_{Fl}^+\right) \quad (71)$$

This is an interesting result because it provides a relationship between DSS terms, the DSS effect parameter and action with well-known dimensionless quantities, the Richardson number, flow resistance number, loop characteristic length number, and the loop time constant. Additionally, Equation (71) explicitly shows that the DSS effect parameter accounts for the evolution of the agent of change over time.

A major difference between DSS and previous methodologies is that the DSS methodology can also be used to categorize processes. For example, it can be shown that quasi steady-state single-phase natural circulation can be categorized as a constant temporal displacement rate process with zero Gaussian curvature [11]. As a result, the natural circulation process metric coefficients, metric equation, Christoffel symbols, model and prototype geodesics and geodesic separations (i.e., time-dependent distortions) can be readily obtained and evaluated. This geometric nomenclature may be unfamiliar to experiment designers, however this approach opens new areas of exploration in experiment scale optimization and code assessment.

3. CONCLUSIONS

The Dynamical System Scaling (DSS) methodology is based on a differential geometry formalism where a physical process is treated in terms of a geometric object. As stated in [5] the idea of describing physical processes in terms of geometric objects is not new, however this is a novel approach in the thermal-hydraulic community and therefore it is important to provide a relationship with more traditional methods such that benefits can be more readily appreciated by the analysts.

The analysis presented in this paper shows that it is possible to draw a simple parallel between the innovative Dynamical System Scaling (DSS) methodology and more established scaling analysis methodologies in the thermal-hydraulic community such as the Hierarchical Two-Tiered

Scaling (H2TS) and the Fractional Scaling Analysis (FSA) methodologies. DSS offers similarity criteria and a measure of scale distortion which captures the dynamics of the system and can therefore change through time.

The paper provided comparisons with generalized balance equations. Then the scaling methods were compared for a single-phase natural circulation scenario. The analysis of the natural circulation problem provides an interesting result. The DSS terms, the DSS effect parameter and action can be related to well-known dimensionless quantities, the Richardson number, flow resistance number, loop characteristic length number, and the loop time constant. Additionally, it is shown that the DSS effect parameter accounts for the evolution of the agent of change over time, a valuable contribution to existing scaling methodologies.

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