THE DYNAMICAL SYSTEM SCALING METHODOLOGY

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Abstract

This paper introduces the Dynamical System Scaling (DSS) methodology. The DSS method presented herein provides an approach that can be used to optimize process scaling and to assess process scale distortions over the entire duration of a process. This paper introduces the concept of temporal displacement rate and presents the derivation of a process 1-metric that relates reference time to process time. The DSS method describes physical processes as trajectories in a normalized coordinate system typical of that found in the study of dynamical systems. Process similarity, as with geometric similarity, is attributed to an invariance of the process metric under a coordinate transformation. It is shown that a two-parameter affine transformation of the normalized coordinates yields five distinct scaling approaches; including the well-known power-to-volume scaling approach. It is shown how prototype and scaled model processes can be compared and how their relative time-dependent distortions can be quantified using a flat-space approximation in normalized coordinates. The paper concludes with positing research for advanced scaling analytics based on the invariance of a process metric for a 2-dimensional Riemannian space in normalized coordinates.

Keywords: Scaling analysis, dynamical systems, time-dependent scale distortion, H2TS, FSA

1. INTRODUCTION

The Hierarchical Two-Tiered Scaling (H2TS) methodology and more recently the Fractional Scaling Analysis (FSA) methodology are well established techniques that have been adopted in many studies [1] -[7]. To assess time-dependent scaling effects, recent studies [8]-[9] have extended these methods to include dimensionless groups based on the initial conditions, average conditions, or final conditions. A method for assessing time dependent scaling distortions evaluated as the transient evolves is discussed in [10]. The Dynamical System Scaling (DSS) approach presented in this paper is a powerful new methodology that builds from the H2TS and FSA approaches by incorporating the system's dynamic response into the scaling framework. The purpose of this paper is to introduce the underlying theory for the DSS approach as applied to a single dynamical process. Future papers will describe the application of the DSS approach to hierarchical systems.

2. THEORY

The first step in the DSS approach is to describe physical processes in terms of points and arc lengths in a special coordinate system; also known as a *phase space* in the study of dynamical systems [11]. The idea of describing physical processes in terms of geometric objects is not new. Klein's (1893) Erlangen program proposed that physical processes can be described in terms of the invariant properties of geometric objects [12]. By describing processes in terms of geometry, this paper proposes that the principles of geometric similarity can be used to assess process similarity. In the DSS approach, process similarity, as with geometric similarity, can simply be viewed as invariance under a coordinate transformation. This section introduces the underlying mathematics and methods needed to a) relate an externally measured reference time scale (i.e., clock) to the "natural" often non-linear time scale of a process, and b) describe the evolution of a process as a trajectory on a surface defined in terms of process coordinates and parameterized by process time.

2.1 Definitions and Fundamental Concepts

A system is defined as a finite control volume containing a conserved quantity such as mass, momentum, or energy and acted upon by internal and external agents of change. A process is defined as the sequential transition of the state of the system; the transition sequence governed by an integral system balance law constrained by the system's initial state and boundary conditions.

Reference time is as defined by Einstein and Infeld [13] which recognizes a clock as a special type of process; one capable of producing constant time intervals. A reference time coordinate is constructed by adding the constant time intervals produced by such a process. The time scale produced by a reference clock is defined as the t-coordinate in this paper. Reference clocks have no physical relationship to the process being measured. They can only provide a natural parameterization for physical processes evolving with constant time intervals. Reference time shall later be shown to be a subset of process time.

2.1.1 Integral Balance Equation

Consider an arbitrary control volume containing a conserved quantity such as a specific mass, momentum, or energy that is a function of position $\vec{x}$ within the control volume and that varies relative to an external reference time measurement, *t*. It is assumed that the spatial and temporal coordinates are continuous. It is further assumed that the control volume is bounded by a control surface that is deformable, has a local instantaneous surface velocity $\vec{v}_s$, and has an outward normal vector $\vec{n}$. Let the local and instantaneous amount of a conserved quantity distributed within the system be denoted by $\psi(\vec{x},t)$. This quantity can change by material transport $\psi(\vec{v}-\vec{v}_s)\cdot\vec{n}$ into or out of the control volume (e.g., fluid enthalpy transport) or it can change as a result of a flux $(\vec{j}\cdot\vec{n})$ applied at the surface (e.g., surface heat transfer). It can also change as a result of volumetric sources or sinks ϕ_v that produce or deplete the conserved quantity. Finally, the amount of conserved quantity can change as the result of the action of external fields ϕ_f, such as gravitational, magnetic, or electric fields. Taking into account all of these *agents of change*, φ_i, including multiple fluxes and material transports (summed from i=1 to n), the integral balance equation for the system is written as:

$$\frac{d}{dt}\iiint_V \psi(\vec{x},t)\,dV = \iiint_V \left(\phi_v + \phi_f\right)dV + \iint_A \left(\vec{j}\cdot\vec{n}\right)dA - \iint_A \psi\left(\vec{v}-\vec{v}_s\right)\cdot\vec{n}\,dA = \sum_{i=1}^{n}\varphi_i\,. \tag{1}$$

The following definitions are now introduced to aid in later sections. First, the normalized integral amount of conserved quantity at a given instant is defined as follows:

$$\beta(t) = \frac{1}{\Psi_o}\iiint_V \psi(\vec{x},t)\,dV\,. \tag{2}$$

In this equation, Ψ_o is a time-independent (ideally maximum) value of the integrated conserved quantity for the process considered. Furthermore, Ψ_o, can also be expressed as a maximum interval relative to a reference datum. Next, the parameter $\omega(t)$ shall be defined as:

$$\omega(t) = \frac{1}{\Psi_o}\left[\iiint_V \left(\phi_v + \phi_f\right)dV + \iint_A \left(\vec{j}\cdot\vec{n}\right)dA - \iint_A \psi\left(\vec{v}-\vec{v}_s\right)\cdot\vec{n}\,dA\right] = \frac{1}{\Psi_o}\sum_{i=1}^{n}\varphi_i \tag{3}$$

This parameter has units of inverse time and is proportional to the sum of the agents of change. Substituting Eqs.(2) and (3) into Eq. (1) yields the integral balance equation in terms of temporal units as follows:

$$\frac{d\beta}{dt} = \omega\,. \tag{4}$$

2.1.2 Process Time as a Natural Parameter

The process time is defined as follows:

$$\tau = \frac{\beta}{\omega}\,. \tag{5}$$

Equation (5) is the process time defined by Zuber in 2001 [14]. He defines the process time as the inverse of the fractional rate of change of a conserved quantity. The results obtained in his study are quite insightful and have found excellent use in phenomenon scaling. Other researchers have considered similar definitions [15]- [18]. Because the sum of the agents of change can be positive or negative, the process timeline can run in either the positive or negative direction.

2.1.3 The Process Time Transformation Law

Taking the derivative of the process time Eq. (5), with respect to the reference time, t, gives the following result:

$$\frac{d\tau}{dt} = \frac{1}{\omega}\frac{d\beta}{dt} - \frac{\beta}{\omega^2}\frac{d\omega}{dt}\,. \tag{6}$$

It is immediately recognized that the first term on the right side of Eq. (6) is unity by virtue of Eq. (4). Thus, Eq. (6) becomes:

$$\frac{d\tau}{dt} = 1 - \frac{\beta}{\omega^2}\frac{d\omega}{dt}. \tag{7}$$

A useful insight into the process time transformation law can be obtained by defining a *temporal displacement rate, D,* as follows:

$$D = \frac{d\tau - dt}{dt}. \tag{8}$$

This is analogous to the elongation rate of a material undergoing deformation (see for example Dym and Shames) [19]. Substituting Eq. (8) into Eq. (7) reveals:

$$D = -\frac{\beta}{\omega^2}\frac{d\omega}{dt}. \tag{9}$$

Hence, the final form of the transformation law in differential form shall be expressed as:

$$d\tau = \left(1 + D\right)dt. \tag{10}$$

Equation (10) is a *1-metric* that relates the infinitesimal process time interval to the infinitesimal reference time interval. For $D > 0$, the process time interval is dilated relative to the reference time interval. Conversely, for $D < 0$, the process time interval is contracted relative to the reference time interval. Accelerated processes are defined as those processes having a nonzero value of *D*.

The transformation law for the process time interval is obtained by integrating Eq.(10) as follows:

$$\tau_2 - \tau_1 = \int_{t_1}^{t_2} \left(1 + D\right)dt \tag{11}$$

For a constant rate process, the sum of the agents of change, $\sum_i^n \varphi_i$, is a constant. Subsequently *D* is equal to zero because $d\omega/dt$ is equal to zero. Applying $D = 0$ to Eq. (11) yields the result that the process time interval and the reference time interval are equivalent (i.e., $\Delta\tau = \Delta t$). This result agrees with Einstein's definition of a clock being based on constant rate processes. Equation (11) reveals that *reference time is a subset of process time*. Therefore a reference clock is any process having a value of *D* equal to zero.

In summary, the definition proposed for process time, Eq.(5), unifies the concept of an externally measured reference time scale and the time scale that arises from the change in the conserved quantity within a system. Furthermore, Eq. (11) indicates that all process time scales are relative. That is, every process creates a time scale that can be related to any reference time scale, and by extension, to any other process time scale, by their process specific temporal displacement rates.

2.2 Describing Processes in Phase Space (Process Space-time)

The instantaneous integral state of a system shall be defined as a point on a $\beta-\omega$ coordinate phase plane. The process evolves along a trajectory in phase space parameterized by lines of constant τ. The τ-coordinate lines are labelled as the null geodesics of the surface because $d\tau = 0$ along the τ-coordinate line. Similarly, the process trajectory shall be labelled as the process geodesic. These labels are strictly true only for the case of a phase space defined by independent coordinates. This dynamical phase space can be viewed as a *process space-time*. It is important to note that the state of a system is defined both in terms of the conserved quantity and the agents of change.

The depiction of process curves on the $\beta-\omega$ plane is directly related to the concept of a phase portrait for a dynamical system. This well-known technique provides powerful qualitative insights into the stability, classification, and topological equivalence of system behaviors [11]. Plotting process curves in $\beta-\omega-\tau$ space-time introduces the quantitative power afforded by measuring the arc-length of a system state trajectory, or flow, using the process time scale, τ. Figure 1 depicts a process curve in $\beta-\omega-\tau$ space-time for the purpose of illustrating the concepts.

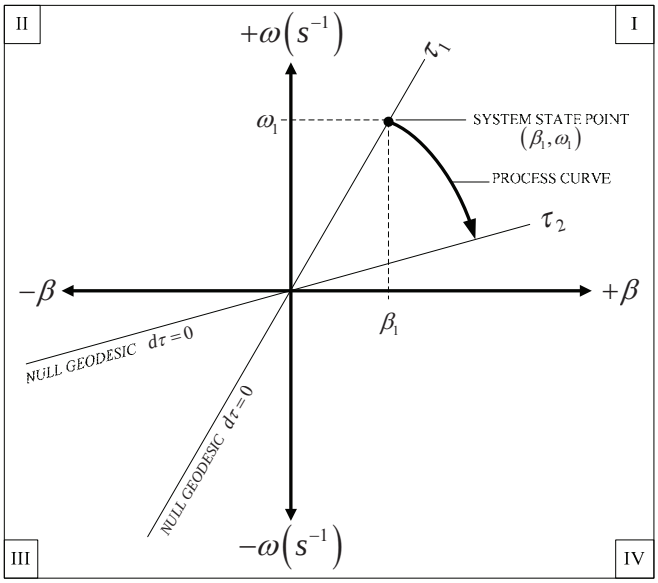

Figure 1. Phase curve for a process depicted in the $\beta-\omega$ parameter plane with τ as the process time coordinate

The process trajectory is represented by the curve originating at the state point labeled τ_1 and extending to τ_2. The null geodesics passing through the origin is the process time coordinate. The arc length for any portion of the process curve can be determined in each of the four quadrants. There is no requirement that the process space-time surface itself be everywhere flat. That is, the surface can be curved and possess intrinsic geometric properties of value to the

analysis. As such, a complete geometric analysis of a process requires a *2-metric* equation for measurements on curved surfaces which is the subject of future research.

3. Process Similarity Governed by Metric Invariance

The action or arclength of the process trajectory shown in Figure 1 shall be defined as follows:

$$\tau_S = \int_{t_1}^{t_2} (1+D)\, dt\,. \tag{12}$$

It is proposed that the action, τ_S, given by Eq (12) serve as the normalization factor for the process 1-metric. This selection is based on requiring that the integral of the normalized 1-metric equal unity as follows:

$$\int_{t_i}^{t_f} \frac{d\tau}{\tau_S} = \frac{1}{\tau_S}\int_{t_i}^{t_f} (1+D)\, dt = \int_{t_i}^{t_f} d\tilde{\tau} = 1 \tag{13}$$

The normalized coordinates and parameters are then defined as:

$$\tilde{\Omega} = \omega\tau_S\,; \qquad \tilde{\beta} = \beta; \qquad \tilde{t} = \frac{t}{\tau_S}; \qquad \tilde{\tau} = \frac{\tau}{\tau_S} \tag{14 a-d}$$

Equations (14 a-d) introduce an effect parameter, $\tilde{\Omega}$. It is recognized as Zuber's *effect metric* except that it has been normalized by the process action which is an integral over the entire duration of the process. The descriptive term "parameter" is used to distinguish it from the formal concept of a "metric" as defined in differential geometry.

Applying the normalized coordinates to Eq. (9) reveals that the normalized temporal displacement rate is invariant under this coordinate transformation. That is, τ_S cancels such that:

$$\tilde{D} = -\frac{\tilde{\beta}}{\tilde{\Omega}^2}\frac{d\tilde{\Omega}}{d\tilde{t}} = -\frac{\beta}{\omega^2}\frac{d\omega}{dt} = D \tag{15}$$

The normalized 1-metric for the process becomes:

$$d\tilde{\tau} = \left(1 + \tilde{D}\right) d\tilde{t} \,. \tag{16}$$

4. Scaling Methods for a Single Dynamical Process

This section demonstrates how the invariance of the normalized 1-metric, Eq.(16), under different types of coordinate transformations, when coupled to the principle of covariance yields a variety of scaling methodologies.

4.1 Principle of Covariance

Consider a conserved process governed by Eq. (4) and measured using a reference clock with a constant time interval dt_P. Consider the same process measured in a scaled experiment using a different reference clock with a constant time interval dt_M that is linearly proportional to dt_P. The principle of covariance asserts that the ratio $(1/\omega)\, d\beta/dt$ is equal to unity in both reference time frames. That is,

$$\frac{1}{\omega_M}\frac{d\beta_M}{dt_M} = \frac{1}{\omega_P}\frac{d\beta_P}{dt_P} \tag{17}$$

4.2 Normalized Balance Equation

Expanding Eq.(4), the integral balance equations for the prototype (k=P) and the model (k=M) can be written as follows:

$$\frac{d\beta_k}{dt_k} = \sum_{i=1}^{n} \omega_{ik} \tag{18}$$

This equation can be made dimensionless by multiplying both sides of the equation by the process action τ_S to obtain:

$$\frac{d\beta_k}{d\tilde{t}_k} = \tilde{\Omega}_k \tag{19}$$

The effect parameter, $\tilde{\Omega}_K$, is expressed as the sum of the products of the specific frequency for an agent of change and the process action, τ_{SK}. That is,

$$\tilde{\Omega}_k = \sum_{i=1}^{n} \omega_{ik}\tau_{Sk} = \sum_{i=1}^{n} \tilde{\Omega}_{ik} \tag{20}$$

Each of the effect parameters requires a separate equation that describes the local transport phenomena that acts to add or remove conserved quantity to the system. Having defined the governing equation, Eq.(19), for a full-scale prototype (k=P) and an equivalent model

dynamical system (k=M), the following section will demonstrate how a two-parameter affine transformation yields five potential scaling methods for a single dynamical process.

4.3 Transformation of Process Coordinates

Consider a two-parameter affine transformation (without translational components) of the β and ω coordinates as follows:

$$\beta_M = \lambda_A \beta_P \tag{21}$$

$$\omega_M = \lambda_B \omega_P \tag{22}$$

In these equations, the transformation parameters λ_A and λ_B are constant scale factors. This simple coordinate transformation leads to several important results, which will be demonstrated in the next sections. This includes providing the scaling ratio for the reference time in the model to that of the prototype and demonstrating that the temporal displacement rate of a process, *D*, is invariant under affine transformations. This later result is important because the temporal displacement rate embodies the acceleration of a process which is a central feature of scaling dynamical processes. Furthermore, applying the two-parameter transformation to the normalized process metric provides the scaling relationships for the effect parameters. Of major significance is the fact that each combination of one- and two-parameter transformations results in a unique scaling approach.

4.3.1 Reference Time Scaling and the Invariance of the Temporal Displacement Rate

The scaling relationship for reference time in the model relative to that in the prototype is obtained by substituting the transformation parameters, Eq. (21) and Eq.(22), into the covariance principle described by Eq.(17). This yields the simple scaling relationship:

$$\frac{\lambda_A}{\lambda_B} = \frac{dt_M}{dt_P} \tag{23}$$

Similarly, substituting Eqs.(21) and (22) into Eq. (9) results in the following expression relating the model and prototype temporal displacement rates:

$$D_M = -\frac{\beta_M}{\omega_M^2}\frac{d\omega_M}{dt_M} = -\frac{\beta_P}{\omega_P^2}\frac{d\omega_P}{dt_P}\left(\frac{\lambda_A}{\lambda_B}\frac{dt_P}{dt_M}\right) \tag{24}$$

Substituting the definition of the prototype temporal displacement rate into Eq. (24) yields:

$$D_M = D_P\left(\frac{\lambda_A}{\lambda_B}\frac{dt_P}{dt_M}\right) \tag{25}$$

Finally, substituting the reference time scaling ratio, Eq.(23), into Eq. (25) yields the desired demonstration that the temporal displacement rate is invariant with a two parameter affine transformation of coordinates. That is,

$$D_M = D_P \tag{26}$$

4.3.2 Invariance of the Normalized 1-Metric

The 1-metric for the model is defined using Eq. (16) as follows:

$$d\tilde{\tau}_M = \left(1 + D_M\right) d\tilde{t}_M \tag{27}$$

Substituting Eqs. (25) and (26) into Eq. (27), yields:

$$d\tilde{\tau}_M = d\tilde{\tau}_P \left[\frac{\lambda_A \tau_{SP}}{\lambda_B \tau_{SM}} \right] \tag{28}$$

For the 1-metric to be invariant under the specified coordinate transformation, the process action scaling ratio must scale as follows:

$$\frac{\tau_{SM}}{\tau_{SP}} = \frac{\lambda_A}{\lambda_B} \tag{29}$$

As such, the process action scales as the reference time. Applying this requirement to (28) yields the 1-metric invariance:

$$d\tilde{\tau}_M = d\tilde{\tau}_P \tag{30}$$

4.3.3 Effect Parameter Scaling Ratios

The integral balance equation for the model can be written as follows:

$$\tau_{SM} \frac{d\beta_M}{dt_M} = \tilde{\Omega}_M \tag{31}$$

Similarly, the integral balance equation for the prototype is given by:

$$\tau_{SP} \frac{d\beta_P}{dt_P} = \tilde{\Omega}_P \tag{32}$$

Substituting Eq.(21), Eq.(22), and Eq. (25), into Eq. (26) yields:

$$\lambda_A \tau_{SP} \frac{d\beta_P}{dt_P} = \tilde{\Omega}_M \tag{33}$$

Dividing Eq. (33) by Eq. (32) yields the following scaling ratio for the effect parameter:

$$\lambda_A = \frac{\tilde{\Omega}_M}{\tilde{\Omega}_P} \tag{34}$$

Expanding Eq. (34) using Eq. (20) yields the following ratio:

$$\lambda_A = \frac{\tilde{\Omega}_{1,M} + \tilde{\Omega}_{2,M} + ... + \tilde{\Omega}_{n,M}}{\tilde{\Omega}_{1,P} + \tilde{\Omega}_{2,P} + ... + \tilde{\Omega}_{n,P}} \tag{35}$$

The solution to Eq. (35) is found by scaling each term individually such that:

$$\lambda_A = \frac{\tilde{\Omega}_{1,M}}{\tilde{\Omega}_{1,P}};\ \lambda_A = \frac{\tilde{\Omega}_{2,M}}{\tilde{\Omega}_{2,P}};\ \lambda_A = \frac{\tilde{\Omega}_{3,M}}{\tilde{\Omega}_{3,P}};... \lambda_A = \frac{\tilde{\Omega}_{n,M}}{\tilde{\Omega}_{n,P}} \tag{36}$$

Eq. (36) provides the scaling ratios for the individual effect parameters of a single dynamical process.

4.3.4 Similarity Criteria and Scaling Approaches

Table 4-1 lists the five different combinations of constant scale factor (λ_A, λ_B) transformations that have been applied to the prototype β and ω coordinates. Each combination results in a different scaling method with a unique set of scaling criteria. All of the scaling methods preserve the invariance of the normalized process metric under transformation. That is, $\tilde{\tau}_R = 1$ for all of the scaling methods even though the reference time-scale in the model may be dilated or contracted relative to that in the prototype.

Table 4-1 Scaling Methods and Similarity Criteria Resulting from Two-Parameter Transformations

Basis for Process Space-time Coordinate Scaling				
Metric Invariance	$d\tilde{\tau}_P = d\tilde{\tau}_M$	And	Covariance Principle	$\frac{1}{\omega_P}\frac{d\beta_P}{dt_P} = \frac{1}{\omega_M}\frac{d\beta_M}{dt_M}$
$\beta - \omega$ Coordinate Transformations				
2-2 Affine $\beta_R = \lambda_A;\ \omega_R = \lambda_B$	Dilation $\beta_R = \lambda;\ \omega_R = \lambda$	β - Strain $\beta_R = \lambda;\ \omega_R = 1$	ω - Strain $\beta_R = 1;\ \omega_R = \lambda$	Identity $\beta_R = 1;\ \omega_R = 1$
Similarity Criteria				
$\tilde{\Omega}_R = \lambda_A$ $\tau_R = t_R = \frac{\lambda_A}{\lambda_B}$	$\tilde{\Omega}_R = \lambda$ $\tau_R = t_R = 1$	$\tilde{\Omega}_R = \lambda$ $\tau_R = t_R = \lambda$	$\tilde{\Omega}_R = 1$ $\tau_R = t_R = \frac{1}{\lambda}$	$\tilde{\Omega}_R = 1$ $\tau_R = t_R = 1$

The five scaling methods are subsets of a two-parameter general affine transformation in the parameter plane. The first scaling method is based on a two-parameter, two-coordinate affine transformation (*2-2 affine transformation*) without translational components. The 2-2 affine transformations do not preserve distances or angles. However, they do preserve parallelism. In this case, prototype geodesics would be parallel to model geodesics in the transformed space.

The second scaling method is based on the *dilation transformation*, which is a similarity transformation without translational components. A similarity transformation is a one-to-one mapping of the plane onto itself such that each distance is multiplied by a single constant scale factor. Although it is typically called a dilation transformation, distances can be dilated or contracted. Angles and shapes remain invariant under a dilation transformation.

The third and fourth scaling methods are based on strain transformations. They are essentially unidirectional dilation transformations. The β*-strain transformation* applies a constant scale factor, λ_A, to the β-coordinate while leaving the ω-coordinate unchanged. Conversely, the ω*-strain transformation* applies a constant scale factor, λ_B, to the ω-coordinate while leaving the β -coordinate unchanged. The scaling analyses performed for the Advanced Plant Experiment (APEX) for the AP600 and AP1000 test programs used a one-half time scaling approach that can be shown to be an ω*-strain transformation* of the coordinates. [1]

The last scaling method is based on the *identity transformation.* The identity transformation preserves distances between points in the prototype coordinates and the model coordinates. Distances and angles are invariant under an identity transformation. Many of the legacy nuclear test facilities described in Appendix A of NUREG-1230 were designed using the well-known power-to-volume scaling approach. Power to volume scaling can be shown to be an *identity transformation* of the coordinates.

4.4 Method of Geodesic Separation to Assess Scale Distortion

The DSS method describes processes as finite curves on the $\beta - \tilde{\Omega} - \tilde{\tau}$ surface. The separation between corresponding points on the prototype geodesic and the model geodesic represents the scale distortion at a particular normalized process time, $\tilde{\tau}$. For an ideally scaled process, the prototype geodesic and the model geodesic will overlay exactly when plotted in normalized process space-time. However, any distortion in the scaling process will create a local separation between the two geodesics. Integrating the *geodesic separation* over the entire duration of the process yields a *dynamical assessment* of the scale distortion. This approach is similar to Einstein's study of geodesic deviation to determine the rate at which two inertial bodies traveling along geodesics accelerate towards each other. However, rather than examining the second time derivative of the geodesic separation, the study of scale distortion can be fully implemented using the values of geodesic separation and its first derivative.

For purposes of illustration, let a scale distortion be introduced such that the prototype geodesic and the model geodesics do not overlay in process space-time as shown in Figure 2. To determine the scale distortion, it is necessary to calculate the separation distance, $\eta_\chi(\tilde{\tau})$, between the points $(\beta_P, \tilde{\Omega}_P)$ and $(\beta_M, \tilde{\Omega}_M)$ along the null geodesics as shown in Figure 2. The simplest approach is to consider the metric for a flat space defined as follows:

$$d\eta_\chi^2 = d\beta^2 + d\tilde{\Omega}^2 \tag{37}$$

The subscript χ has been added to denote a flat space. The normalized metric for the null geodesic is given by:

$$0 = d\beta^2 - \tilde{\tau}^2 d\tilde{\Omega}^2 \tag{38}$$

Substituting Eq (38) into Eq. (37) and taking the square root yields the differential expression for the geodesic separation in the parameter plane:

$$d\eta_\chi = \sqrt{1+\tilde{\tau}^2}\, d\tilde{\Omega} \tag{39}$$

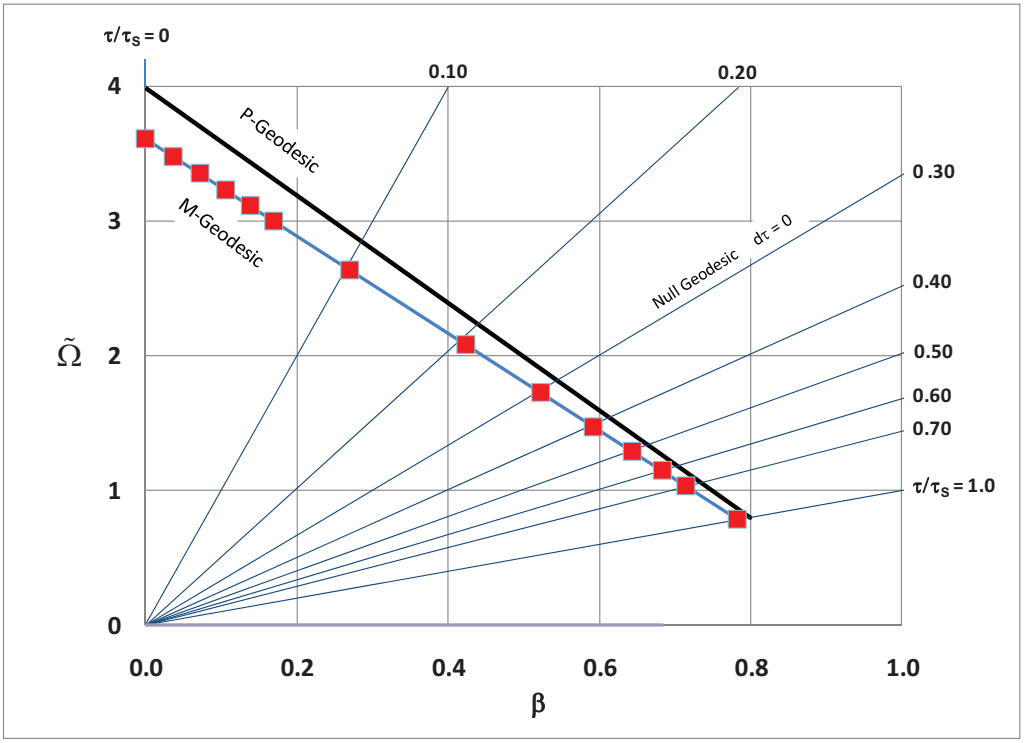

Figure 2 A scale distortion is introduced such that a geodesic separation is created between the model geodesic curve (points) and the prototype geodesic curve (solid line).

Integrating from $\tilde{\Omega}_M$ to $\tilde{\Omega}_P$ yields the desired equation for the geodesic separation for corresponding points on the model and prototype geodesics in the flat parameter plane:

$$\eta_\chi(\tilde{\tau}) = \sqrt{1+\tilde{\tau}^2}\left(\tilde{\Omega}_P - \tilde{\Omega}'_M\right) = \sqrt{1+\tilde{\tau}^2}\left(\tilde{\Omega}_P - \frac{\tilde{\Omega}_M}{\lambda_A}\right) \tag{40}$$

This equation takes into account a second transformation of the model effect parameter when $\lambda_A \neq 1$. Eq. (40) is generally applicable to processes that evolve in flat space. That is, having a

Gaussian curvature of K=0. It measures the local geodesic separation as a function of normalized process time. For this simple example, the geodesic separation becomes increasingly smaller as the model and prototype processes converge towards a common equilibrium point. The total scaling distortion is defined as the integral of the geodesic separation over the process time interval of interest. That is,

$$\eta_T = \frac{1}{\left(\tilde{\tau}_F - \tilde{\tau}_I\right)} \int_{\tilde{\tau}_I}^{\tilde{\tau}_F} \left|\eta_\chi\left(\tilde{\tau}\right)\right| d\tilde{\tau} \tag{41}$$

In this equation, the integral of the *absolute value* of the geodesic separation is implemented to take into account the effect of compensating errors. The integral is evaluated over the normalized process time interval from 0 to 1. For this example, the total scaling distortion is found to be 8.3 percent over the entire duration of the process using the flat space approximation.

The method of geodesic separation provides a dynamic analysis of the scaling distortion for the entire duration of the process. The results for this example demonstrate that a scaling distortion introduced at the onset of a process may not remain the same throughout the process. Therefore, the assessment of a test facility based on a scaling distortion evaluated for a single set of conditions may not fully characterize the scaling distortion for the entire process. It may be overly conservative, as in this example, for processes that converge towards a common equilibrium point. It is equally important to identify processes for which the introduction of a scale distortion leads to M-geodesics and P-geodesics that diverge.

5. Future Research

The description thus far has been limited to the application of the DSS method to a single dynamical process using a 1-metric. It is recognized that many practical applications require scaling of integral systems with multiple components and coupled processes. The application of the DSS method to integral system scaling has been developed and shall be discussed in a future paper.

It was noted in Section 2.2 that there is no requirement that the process space-time surface shown in Figure 1 be everywhere flat. That is, the surface can be curved and possess intrinsic geometric properties such as the Gaussian curvature that could be used to categorize classes of processes or provide new insights into the scaling of processes. Research is needed to develop a Riemannian *2-metric* equation of the form $d\tilde{\tau}^2 = \tilde{g}_{ij} d\tilde{x}^i d\tilde{x}^j$ that is invariant with respect to coordinate transformations. In this equation, $\tilde{g}_{ij}$ is the set of normalized metric coefficients, $\tilde{x}^i$ are the normalized coordinates, and the standard Einstein summation convention for repeated indices has been applied. The outcome of such research would represent the next major advancement in the analysis and scaling of dynamical systems.

6. Conclusions

This introductory paper has demonstrated that the DSS method can be used to scale dynamical processes using the invariance of a simple 1-metric equation under coordinate transformations. It

was shown that a two-parameter affine transformation of the normalized coordinates yields five distinct scaling approaches; including the well-known power-to-volume scaling approach. It was shown how prototype and scaled model processes can be compared and how their relative time-dependent distortions can be quantified using a flat-space approximation in normalized coordinates. Three application papers have been published [21] - [23] to compare the DSS method to existing scaling methods, to demonstrate how the DSS method can be used for test facility optimization and for data synthesis.

7. References

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