Natural convective heat transfer from a heated slender vertical tube in a cylindrical tank

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Abstract

Natural convective heat transfer in enclosures is widely researched in extensive range of engineering application such as passive residual heat removal system (PRHRS) in nuclear plant, solar collectors and cooling of electronic equipment. After Fukushima incident, the application and researching of PRHRS have been gaining more and more attention. It still has no proper calculating tools to analyze natural convective heat transfer with large L/D ratio and high Rayleigh number in PRHRS. In this paper, based on some former natural convective heat transfer experiments, numerical simulations and scale analysis, the single phase natural convective heat transfer correlation from the vertical tube have been developed.

Transient single phase natural convection heat transfer along vertical tube in a cylindrical enclosure has been investigated for water with widely used computational fluid dynamic (CFD) engineering program ANSYS CFX-12. The influence brought by different geometry of tube and mesh scales has been evaluated by the results of former experiments. The ratio of tube length to diameter is a dimensionless factor affects heat transfer rate. The evaluation of thermal stratification to natural convective heat transfer in enclosure has been studied. Numerical simulation covers large L/D and high Rayleigh number natural convection: $10<L/D<300$, $1.34\times10^{8}<Ra_L<1.45\times10^{14}$.

Based on the scale analysis of natural convection cylindrical boundary equations the new dimensionless group has been put forward contains L/D factor. The new natural convective heat transfer along the vertical single tube model has been build according to the former experiments` results. After combining numerical simulation results, the new model covers the higher Rayleigh number range of natural convection.

Keywords: High Rayleigh number, Natural convective heat transfer, L/D, CFX

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Nomenclature

D	tube diameter
C_{pl}	specific heat at constant pressure
g	gravitational constant
h	heat transfer coefficient
L	tube length
Gr_L	average Grashof number $g\beta\Delta TL^3/\nu^2$
Nu_D	average Nusselt number hD/λ
Nu_L	average Nusselt number hL/λ
Nu_x	local Nusselt number hx/λ
Pr	Prandtl number
Q	heat flux
Ra_D	average Rayleigh number $\rho g\beta\Delta TD^3Cpl/\lambda\nu$
Ra_L	average Rayleigh number $\rho g\beta\Delta TL^3Cpl/\lambda\nu$
Ra_x	local Rayleigh number $\rho g\beta\Delta Tx^3Cpl/\lambda\nu$
Ra_x*	modified local Rayleigh number $\rho g\beta Qx^4Cpl/\lambda^2\nu$
T	temperature
t_{avg}	volume averaged temperature
t_∞	bulk temperature
ρ	density
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
β	isobaric coefficient of thermal expansion
ΔT	temperature difference

1. Introduction

Natural convective heat transfer from vertical surface to water has been studied extensively. The experiments and numerical simulations have been developed with different geometric conditions and thermal boundary conditions. A plenty of natural convective heat transfer models are suggested to calculate the Nusselt numbers for vertical plat and vertical tube. But most of these models are correlated by natural convection experiments of low Rayleigh numbers and without special treatment to curvature effect to heat transfer. When face to the natural convective heat transfer from tubes of heat exchanger in PRHRS, the new heat transfer model should cover the range of high Rayleigh numbers and can be applied to slender tube.

In many papers and text books, when calculate the Nusselt numbers of natural convective heat transfer from vertical tube there is a highly recommended limitation equation to judge if the former correlation can be applied without special modification of the curvature of tube within 5% error. This limitation was derived by Sparrow and Gregg [1] in 1956.

$$\frac{d}{H} \geq \frac{35}{Gr_L^{1/4}} \qquad (1)$$

However, when the diameter of tube is small that the Grashof number do not meet the limitation, it has no proper heat transfer correlation recommended.

W. H. McAdams [2] developed a widely used natural convective heat transfer correlation with experiments data of vertical plat and large diameter tube based on the scale analysis of vertical plat boundary equations. Some assumptions were adopted to build the model, such as Boussinesq approximation and boundary layer approximation. The widely used equations are follows:

$$Nu_H = 0.148 Ra_H^{0.333} \quad 10^{10} < Ra_H < 10^{11} \qquad (2)$$

$$Nu_H = 0.48 Ra_H^{0.25} \quad 10^{4} < Ra_H < 10^{9} \qquad (3)$$

After that, many investigators reported natural convective heat transfer models correlated in the same way with McAdams`. To explore the differences of natural convective heat transfer from surface of slender tube and plat, a few researchers focused on numerical solving of boundary layer equation with the same assumption above.

Le Fevre and Ede [3] used the integral equations to study the natural convective heat transfer from vertical tube with the estimations of velocity and temperature parabolic profiles from the analysis of the laminar boundary layer on the vertical flat plate. The average laminar Nusselt number for a vertical cylinder is given by

$$\mathrm{Nu}_{\mathrm{H}} = \frac{4}{3} Ra_H^{0.25} \left[\frac{7\,\mathrm{Pr}}{100 + 105\,\mathrm{Pr}} \right]^{0.25} + \frac{4}{35} \frac{272 + 315\,\mathrm{Pr}}{64 + 63\,\mathrm{Pr}} \frac{H}{D} \tag{4}$$

On the basis of scale analysis of cylindrical boundary layer equations, S. M. Yang [4] suggested that length to diameter ratio should be contained in dimensionless group to build natural convective heat transfer from slender cylinder. A general correlating equation recommended by Yang for laminar and turbulent regions.

$$Nu_H = \left\{ 0.60 \left(\frac{D}{L} \right)^{0.5} + 0.387 \left[\frac{Ra_H}{\left[1 + \left(0.492/\mathrm{Pr} \right)^{9/16} \right]^{16/9}} \right]^{1/6} \right\}^2 \tag{5}$$

H. R. Lee [5] cast the governing boundary layer equations into a dimensionless form by a similar transformation and solved by a finite difference method. As a result, correlation equations for the local and average Nusselt numbers are given when the surface temperature profile satisfy $T_w(x)=T_\infty+ax^n$, but no experiments results were adopted to verify the correlation.

$$\begin{aligned}
&\ln\left[\overline{Nu} \left(G_L / 4 \right)^{-1/4} \right] = R_2(\xi_L) + \left\{ \ln\left[\left(\overline{Nu},_p \right)_{UWT} \left(Gr_L / 4 \right)^{-1/4} \right] - R_2(0) \right\} \cdot \exp\left[- p_2 \xi_L^{1/2} \right] \\
&R_2(\xi_L) = -2.9262 + 1.6685 \xi_L^{1/2} - 0.21909 \xi_L + 0.011308 \xi_L^{3/2} \\
&p_2 = 0.29369 + 0.32635\,\mathrm{Pr}^{-0.19305} \\
&\left(\overline{Nu},_p \right)_{UWT} \left(Gr_L / 4 \right)^{-1/4} = \left(2\,\mathrm{Pr} \right)^{1/2} \left[2.5 \left(1 + 2\,\mathrm{Pr}^{1/2} + 2\,\mathrm{Pr} \right) \right]^{-1/4} \\
&\xi_L = \left(2L / r_0 \right) \left(Gr_L / 4 \right)^{-1/4}
\end{aligned} \tag{6}$$

An experimental study of the laminar free convective average heat transfer in air from isothermal vertical slender tube was performed by C.O. Popiel [6] using a transient technique. The results obtained by experiment for Prandtl number Pr=0.71 in the range of Rayleigh number $1.5\times10^8 < Ra_H < 1.1\times10^9$ were correlated with the equation below.

$$
\begin{aligned}
&Nu_H = ARa_H^n \\
&A = 0.519 + 0.03454(H/D) + 0.0008772(H/D)^2 + 8.855\times10^{-6}(H/D)^3 \qquad (7) \\
&n = 0.25 - 0.00253(H/D) + 1.152\times10^{-5}(H/D)^2
\end{aligned}
$$

This correlation agree well with the known numerical data of Cebeci [7] for fluids of Prandtl number within $0.01<Pr<100$.

Typically, it is hard to acquire high quality experiments data of natural convection heat transfer from slender tube especially to water. In the year 1968, Fuji [8] published experimental investigations on natural convection heat transfer from the outer surface of a vertical cylinder to water. Local heat transfer coefficients for the boundary layer develops through laminar to transition-turbulent and turbulent flow pattern were measured directly. “Quasi-steady state” was defined experimentally as a state equivalent to a steady state with respect to heat transfer coefficients.

Later, H. R. Nagendra [9] investigated natural convection heat transfer from vertical long tube and wires with high length-to-diameter ratio in a quiescent medium of water experimentally. The experiments data were correlated by a dimensionless equation contains the dimensionless group L/D. Only natural convection of laminar flow patternn included in the experiments.

In an attempt to study the effect of tube diameter and turbulent flow pattern to the heat transfer coefficient, Fumiyoshi Kimura [10] carried out several sets of experiments with different diameter of tube in a large tank. The experiments data reported demonstrate that the local heat transfer coefficient in the turbulent flow pattern region is higher than in laminar region. Due to the lack of the heat transfer data, in particular, in the region of the higher Rayleigh numbers, investigators had not obtained a general correlation for the turbulent heat transfer.

M. Ashad [11] also published experiments data of steady state natural convective heat transfer in the turbulent boundary layer regime. All above researchers just studied steady state natural convective heat transfer in enclosure, while in recent years the researchers have extended their studies to transient state natural convection in enclosure with numerical simulation. Arijit A. Ganguli [12] investigated transient natural convection in a cylindrical enclosure with CFD simulations and flow visualization. The effect of various parameters like pressure, tube diameter and aspect ratio on the extent of thermal stratification has been studied in the range of Rayleigh number: $1.08\times10^{12}\le Ra\le 3.76\times10^{12}$.

In order to figure out the exact region which the former natural convective heat transfer correlations and experiments cannot cover, some important correlations are presented in a same plot with Ra_L as x-coordinate and Nu_L as y-coordinate.

In figure 1, it is apparently that when Rayleigh number is higher than 10^8, the deviations between the former correlations predictions are unacceptable, because the different correlations comes from different experiments with different L/D ratios. However, in present applying, the region which the Rayleigh number beyond the limit is interested, so more work should be done to provide solution to this problem.

In the present study, numerical simulation of transient state heat transfer by natural convection from slender tube in enclosure filled by water has been performed with CFX 12 to find out the effect of tube diameter, tube length and Rayleigh number to heat transfer coefficient on the extent of thermal stratification. The data obtained by simulation appear to be very similar to those reported by Fumiyoshi Kimura. Large length to diameter ratio (L/D) of tube and high Rayleigh number natural convective heat transfer has been simulated to provide data for building correlations. More wide range of L/D and Rayleigh number heat transfer data were obtained in the numerical simulation from tube that without any experiments data: $1.34\times10^8 \leq Ra_L \leq 1.45\times10^{14}$. Meanwhile, according to the results of scale analysis the former experiments data of natural convection from outer surface of slender cylinder to water were correlated with new form of dimensionless group. Moreover, owing to the numerical data in higher Rayleigh number the correlation was extended to more wide range of valid parameters.

2. Numerical procedure

Governing equations

A three-dimensional CFD model using CFX code was developed to analyze the transient heat transfer from slender tube to water tank. The modeling was based on the unsteady Navier-Stokes equations within following assumption:

(1) The fluids are treated as single-phase fluids.

(2) The fluids are incompressible with constant properties evaluated at the reference temperature.

(3) The viscous dissipation is neglected.

(4) The Boussinesq approximation is used to consider buoyancy.

(5) Stress transport (SST) k-ω turbulent model is used.

The Reynolds averaged governing equations for mean fields in a Cartesian coordinate system are as below.

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_i u_j\right) = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\left(\mu + \mu_t\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right) + S_i$$

$$\frac{\partial T}{\partial t} + \frac{\partial u_i T}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\left(\frac{\mu}{\mathrm{Pr}} + \frac{\mu_t}{\mathrm{Pr}_t}\right)\frac{\partial T}{\partial x_j}\right)$$

$$\frac{\partial k}{\partial t} + \frac{\partial u_i k}{\partial x_i} = \frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k + P_{kb} - \beta' \rho k \omega$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial u_i \omega}{\partial x_i} = \frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P_k + P_{\omega b} - \beta\rho\omega^2$$

$$S_x = S_y = 0, S_z = \rho g \beta \theta$$

$$P_k = \mu_t \frac{\partial u_i}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), P_{kb} = \frac{\mu_t}{\rho \sigma_\rho} g \cdot \frac{\partial T_i}{\partial x_j}, \sigma_\rho = 0.9$$

$$P_{\omega b} = \frac{\omega}{k}\left(\left(\alpha + 1\right)\max\left(P_{kb}, 0\right) - P_{kb}\right)$$

$$\mu_t = \rho\frac{\alpha k}{\max\left(\alpha\omega, SF\right)}, S = \sqrt{2 S_{ij} S_{ij}}, S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\right)$$

$$F = \tanh\left(\arg^2\right), \arg = \max\left(\frac{2\sqrt{k}}{\beta' \omega y}, \frac{500\nu}{y^2 w}\right)$$

$$\beta' = 0.09, \alpha = 5/9, \beta = 0.075, \sigma_k = 2, \sigma_\omega = 2$$

Validation

In order to evaluate the validity of CFX in numerical simulation of transient natural convection heat transfer in enclosure, the experiment carried out by Kimura in 2004 has been modeled and the simulation data has confirmed the ability of the code.

The thermal boundary conditions of numerical simulation are similar to that in experiment which is showed in table 1.

Based on the grid independence test results the case of 252000 elements of grid has been chosen with simplification of tank size. Considered that the heat flux on the top of the tank is relatively small, adiabatic wall boundary with free slip condition has been applied in simulation. The comparison between the experiment data and the CFD prediction has been showed in figure 2(a), figure 3(b) and figure 4(c). It is obvious that when it comes to the region where the local Rayleigh number is higher than about 3×10^{13} the local Nusselt numbers obtained from experiments are higher than that from CFD. Also the local Nusselt numbers of CFD prediction are almost the same to the experiments data only with a little underestimation of heat transfer at the region of high local Rayleigh numbers. This difference between the experiments data and CFD prediction can attribute to the underestimation in simulating the turbulent flow with SST k-ω model. In experiment, the vortex near the heated wall can be found breaks into smaller vortices near the bulk region and this fluid behavior is helpful to heat transfer. On the contrary, the above phenomena cannot be found in simulation. So far there is still no very well turbulent model for natural convective heat transfer. It has been proved that of all the models SST k-ω model can describe the turbulent natural convection better especially for near wall boundary treatment. In present investigation the difference above is considered acceptable for conservative calculation.

Heat transfer coefficient under the thermal stratification

When heat tubes are placed vertically in a pool of water, the fluid adjacent to the source gets heated up. Since, density of heated water is low, it flows upward. As a result, stable thermal stratification forms in pool. In order to study the influence of this stratification to natural convective heat transfer, a case for transient natural convective heat transfer lasts 1000 seconds is simulated.

It is showed in figure 3 the stable thermal stratification has formed after 1000 seconds. At the top of the tube, the wall temperature rises slightly along with the time increasing in figure 4. The variation of volume average temperature of water in tank compared with the time is showed in figure 5. It is observed that the increasing rate of water temperature almost holds constant. In order to describe the heat transfer from tube to water, various definitions have been given to the average heat transfer coefficient. Actually, there is no absolutely right definition of average heat transfer coefficient and among all these definitions investigators recommends equation 8 as average heat transfer coefficient of steady natural convective heat transfer.

$$\bar{h}_L{}' = \frac{\int_0^L h_x dx}{L} = \frac{\int_0^L \frac{q}{t_x - t_\infty} dx}{L} \tag{8}$$

t_∞ : bulk fluid temperature

But, when it comes to the transient natural convective heat transfer in enclosure, there is no stable bulk fluid temperature. A new equation is been adopted to definite the average heat transfer coefficient of transient natural convection.

$$\bar{h}_L{}'' = \frac{\int_0^L h'_x dx}{L} = \frac{\int_0^L \frac{q}{t_x - t_{\mathrm{avg}}} dx}{L} \tag{9}$$

t_{avg} : volume average fluid temperature

The volume average fluid temperature t_{avg} increases with an increase in time. Further, set the initial water temperature in tank as t_∞ and the heat transfer coefficients profiles of two different definitions are plot in figure 6. It is obviously that choosing $h^{`}_{L}$ cannot combine the effect of water temperature increasing in heat transfer coefficient equation. On the other hand, $h^{``}_{L}$ can be more easily calculated and the variation with an increase in time is smaller compared with $h^{`}_{L}$, which indicates that $h^{``}_{L}$ is a more proper definition equation for transient natural convection heat transfer. Generally, the correlations of natural convection heat transfer are used in thermal hydraulic codes like RELAP, CATHARE and so on. In these codes, it is convenient to acquire the transient volume average water temperature. So, planting the above heat transfer coefficient in codes is acceptable for transient heat transfer procedure. Thermal stratification brings limiting effect to transient natural convection heat transfer, and this effect can be contained in heat transfer coefficient with simplification by a new equation of it.

Simulation cases

It is hard to acquire the experiments data in high Rayleigh number region especially from slender cylinder to water. So, a number of simulation cases are performed to study the turbulent natural convection heat transfer in cylindrical enclosure and provide the prediction data for building a new heat transfer correlation. Constant wall temperature boundary condition is set to tube wall, and other conditions are same to former verification simulation. Based on the analysis in previous chapter, the heat transfer coefficient varies a little with in a time increase. So just choose the prediction data on 100s. As showed in table 2 and 3, the Rayleigh number vary from 1.94×10^{8} to 1.45×10^{14}, the tube length vary from 0.1m to 3m and the tube diameters are 0.01m and 0.02m.

3. Result and discussion

Scale analysis

For a long, vertical tube with a large L/D ratio, transversal curvature effect cannot be neglect and the governing equations for a steady constant-property natural convective heat transfer with estimations in first chapter in cylindrical coordinates are as below:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right]+g\beta\Delta T$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\lambda}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

Appling the scaling can acquire these dimensionless variables:

$$x'=\frac{x}{L};r'=\frac{r}{R};u'=\frac{u}{U};v'=\frac{v}{V};\theta'=\frac{T-T_\infty}{T_w-T_\infty}$$

Then, dimensionless equations are got below:

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial r'}=\frac{\nu}{UR}\frac{L}{R}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u'}{\partial r'}\right)\right]+\frac{Lg\beta(T_w-T_\infty)}{U^2}\theta'$$

$$u'\frac{\partial \theta'}{\partial x'}+v'\frac{\partial \theta'}{\partial r'}=\frac{1}{UR}\frac{\lambda}{\rho C_p}\frac{L}{R}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial \theta'}{\partial r}\right)\right]$$

$$\frac{hR}{\lambda}\theta' = -\left(\frac{\partial\theta'}{\partial r'}\right)_{r\,r=1}$$

Rearrange these dimensionless group can get equations as follow:

$$\frac{D^3 g\beta\left(T_w - T_\infty\right)}{\upsilon^2}\cdot\upsilon\frac{\rho C_p}{\lambda}\cdot\frac{D}{L} \equiv Gr_D\cdot\Pr\cdot\frac{D}{L} = Ra_D\cdot\frac{D}{L}$$

$$\frac{hD}{\lambda} = Nu_D$$

Considered using the tube length as characteristic length of dimensionless equations, the final equations can be written as follows:

$$Nu_L = \frac{L}{D} f\left(Ra_L^{1/4}\cdot\frac{D}{L}\right) \qquad (10)$$

At the right of this equation, L/D ratio contains two independent variables of geometry scale, which will make correlation based on this equation complicated and hard to achieve a unified form. In order to get a more simple and convenient equation, $Ra_L^{1/4}$ is introduced in to transform the original one to that as follows:

$$\frac{Nu_L}{Ra_L^{1/4}} = \frac{\frac{L}{D} f\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)}{Ra_L^{1/4}} = \frac{f\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)}{Ra_L^{1/4}\frac{D}{L}} = f'\left(Ra_L^{1/4}\cdot\frac{D}{L}\right) \qquad (11)$$

With this equation the transversal curvature effect is covered in L/D ratio. It can be used to build natural convective heat transfer correlation from slender tube to water with large L/D ratio.

Correlated the former experiments

After deeply investigating the natural convective heat transfer from slender tube to water, several experiments with specific heat transfer data are chosen to build a new correlation. The experiments information are showed in table 4 and figure 7.

When set the Ra_L as x-coordinate, Nu_L as y-coordinate in logarithmic coordinate, the experiments data for different L/D ratio distribute widely without a consistent curvilinear trend. So, it is not sufficient to correlate the different geometry condition data with large L/D ratio in an equation which only contains with dimensionless group Ra_L. In the light of the scale analysis, the experiments data are rearranged in logarithmic coordinate with $Ra_L^{1/4}D/L$ as x-coordinate and $Nu_L/Ra_L^{1/4}$ as y-coordinate.

In figure 8 shows distribution of the rearranged experiments data which is the same to data in figure 7. Although it cannot present the specific Rayleigh numbers or the Nusselt numbers of experiments directly, the rearranged experiments data point show a good curvilinear trend. Figure 8 illustrates that the new dimensionless equation gets from scale analysis of governing equations is competent for correlating the experiments data of the natural convective heat transfer from slender tube. In an attempt to acquire a specific correlation which is based on the experiments data, the polynomial fits are done with image tool. As a result, the quadric polynomial correlation is got as follows.

$$Log_{10}\left(\frac{Nu_L}{Ra_L^{1/4}}\right)=0.059-0.464Log_{10}\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)+0.239\left(Log_{10}\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)\right)^2$$

（12）

This is valid for:

$10^8 < Ra_L < 10^{12}, 11.5 < \frac{L}{D} < 500$, water as working fluid

Figure 10 shows the parity plot of experiments data with the predicted result from the dimensionless correlation. The deviation between the correlation prediction and experiments data is within 20%.

When fit the experiments data in cubic polynomial correlation, we get the more complicated and, at the same time, more precious correlation as below:

$$Log_{10}\left(\frac{Nu_L}{Ra_L^{1/4}}\right)=0.090-0.449Log_{10}\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)$$
$$+0.107\left(Log_{10}\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)\right)^2+0.065\left(Log_{10}\left(Ra_L^{1/4}\cdot\frac{D}{L}\right)\right)^3 \quad (13)$$

This is valid for:

$10^8 < Ra_L < 10^{12}, 11.5 < \frac{L}{D} < 500$, water as working fluid

As showed in figure 11, the deviation between the correlation prediction and experiments data decreases to 15%.

Extending the valid range of correlation with CFD prediction

Furthermore, the PRHR cooling tank works in high Rayleigh number region which can reach to 10^{14}. Considered that there are few natural convective heat transfer experiments data in that region, the CFD prediction is adopted to extend the valid range of correlation. The calculation procedure has been described in last chapter. Combined with these CFD predictions, the correlation is illustrated to extend to high Rayleigh number region. Figure 12 shows the CFD predictions in the same plot with the experiments data.

From this plot, the CFD predictions (the red and blue data point) distribute in a profile which is similar with the experiments data. In the same way, the parity plot of the correlation predictions and CFD predictions is got to evaluate the deviation between two of those. Significantly, the deviation between the correlation prediction and CFD predictions is within 22% showed in figure 13, which is a good agreement with the new correlation. And more importantly, the correlation is proved to be valid for an extensive range of Rayleigh number:

$$10^8 < Ra_L < 1.45\times10^{14}, 10 < \frac{L}{D} < 500$$

Because the effects of L/D and Ra are into a synthesis unity, the valid range of correlation should be qualified by new dimensionless group $Ra^{1/4}D/L$. Then the correlation is valid for:

$$0.275 < Ra_L^{1/4} \cdot \frac{D}{L} < 85.015$$

Comparison with other correlations

To explore the differences of predictions between the former correlations and present correlation, the comparison is made in figure 14.

The correlations in this plot are showed in table 5.

Because there is the similar dimensionless group with the present one in Lee correlation, it shows some agreement in a narrow range. It is also proved that Lee correlation is not good enough to predict natural convective heat transfer coefficient from slender tube, although the curvature effect had been considered in this correlation. Meanwhile, the Yang correlation and Le Fevre correlation are given the poor prediction of Nusselt number. As a conclusion, just contained L/D ratio in variable of equations without special treatment with the curvature effect cannot give a good result sufficiently. Apparently, the recommendatory dimensionless equation acquired from the scale analysis above is proved to correlate the experiments and CFD predictions well.

4. Conclusion

In laminar region, CFX 12 with k-ω-SST turbulent model can simulate transient natural convective heat transfer in water preciously and the predictions given by CFD method are consistent with the experiments data. But when it comes to the turbulent region with high Rayleigh number, the CFD predictions of Nusselt number is a little lower than experiments data. Choose the definition of heat transfer coefficient with the average volume temperature replaced the bulk temperature can describe the natural convective heat transfer rationally.

As a result of scale analysis, the curvature effect of slender tube with large L/D ratio should not be ignored. The new dimensionless equation contains new dimensionless group of L/D which can correlate the laminar natural convective experiments data from slender tubes to water with various L/D ratios very well. Due to the lack of turbulent natural convective heat transfer experiments data, CFD predictions are adopted in correlating. Finally, the correlation is extended to larger range of Rayleigh number and L/D ratio. The new correlation is valid for $0.275<Ra^{1/4}D/L<85.015$.

Compared with other correlations like Lee, Yang and Le fevre recommended by other investigators the new correlation can predict the Nusslet number more consistent with the experiments data.

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