# ANALYTICAL STABILITY ANALOGUE FOR A SINGLE-PHASE NATURAL CIRCULATION LOOP

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**ABSTRACT**

Numerical solutions for transient fluid flow in nuclear systems often suffer from the effects of numerical diffusion and damping making the assessment of system stability rather difficult. Efforts for coping with this problem include research and development of algorithms with improved fidelity for stability calculations as they apply to particular problems. Benchmarking exercises in comparison with specially designed experiments are necessary to verify algorithmic fidelity and guide the development and adjustments of the algorithms. In this paper, an analytical approach is introduced where a simple model -- an analogue -- is constructed such that the basic instability mechanisms are represented in a form that lends itself to analytical solutions that are free from the diffusion and damping problems that plague finite volume algorithms. Direct conclusions can be made regarding the stability of a system in the case the analogue closely resembles the system under study. However, when the system is too complex for direct assessment, the stability fidelity of numerical solutions can be assessed by comparing the numerical solution for the simple system with the analytical solution and use the comparison to quantify any damping effects and justify the application of the numerical method to the complex representation of the real system under study.

**KEYWORDS**
Natural Circulation, Stability, Differential-Difference Equations, NuScale Reactor Module

## 1. INTRODUCTION

The NuScale Power reactor design is a small modular PWR. The steam generator is integrated within the reactor vessel and the reactor coolant flow is driven by natural circulation which is an important aspect of its passive design philosophy. The natural circulation buoyancy driving head is created by the density difference between the relatively high temperature flow exiting the core and the lower temperature flow returning through the downcomer annulus where the steam generator is the heat sink. In the absence of a recirculation pump, the natural circulation head is itself dependent on the power level and flow rate, which is a feedback mechanism that may potentially lead to unstable flow oscillations.

The basic mechanism of the instability of the primary circuit flow is delayed feedback of the buoyancy driving head resulting from a flow rate perturbation. Consider initial steady state operation perturbed by an increase in the flow rate, the core exit temperature will drop in proportion to the flow increase and the time delay for the core exit temperature response is rather small as the time it takes a fluid particle to traverse the core is much shorter than the time for circulating the entire primary loop. The effect of the exit core temperature decrease is delayed by the time it takes for the fluid exiting the core to fill the riser, thus creating the maximum impact on the density of the fluid in the riser. This delayed feedback is negative, as it will serve to reduce the buoyancy driving head and cause a flow reduction that opposes the originally assumed perturbation. However, when the delayed feedback is sufficiently strong, the system becomes unstable and the flow undergoes growing oscillations.

The problem of natural circulation stability of single-phase flow has been studied and reported in several publications. However, extensive literature review is outside the intended scope of this work. One interesting example is the work of Pilkhwal et al. [1], where they performed analytical studies on experimental data with flow in a rectangular loop. They found instabilities only in the case of horizontal heater (in the bottom pipe) and horizontal cooler (in the top pipe). Other arrangements such as vertical heater and/or cooler were found to be "always" stable. The finding of always stable is not as strong as "unconditionally" stable as it leaves room for the possibility of instability under different conditions such as friction or operating power beyond the particular experiment. Also, the analytical tools in the literature employ finite volume numerical techniques for which numerical damping is a serious concern. In this work, instead of a rectangular loop with four pipes, the analogue is based on a single vertical pipe where the cold leg is simplified using ideal assumptions; thus the simplest geometry is used. In the analysis, no finite differencing or finite volume numerical techniques are employed, rather analytical closed form solutions are obtained so the issues of numerical damping are avoided. The analytical techniques follow Farawila [2].

## 2. PROBLEM SUMMARY

A simplified analogue for the flow in the main loop of the NuScale module is accessible for analytical solution. The loop geometry consists of a fixed power heater representing the nuclear core, that is short compared to the riser section; the loop is closed via the cold leg downcomer where the heat sink heat exchanger is placed at the top of the cold leg. The simple analogue is idealized such that the heat exchanger is so efficient that the temperature (and density) in the cold leg remain constant regardless of the temperature variations of the fluid coming from the riser section. This idealization can be realized by substituting the closed loop geometry with an open one where the cold leg is substituted by a large tank to impose constant pressure drop boundary condition. A sketch of the closed loop (left) and its open loop idealization (right) is given below in Figure 1.

The analogue developed for the idealized geometry sheds light on the broad properties of the system with regard to its stability without influence of side issues like the dynamical effects of the cooling device. These effects will be addressed in a latter section.

The following is a brief description of the analytical model equations and solution used to provide the proof that the single-phase natural circulation hydrodynamic behavior is unconditionally stable for the idealized system.

The buoyancy driving head, $\Delta P_{boyancy}$, is the fundamental mechanism inducing single phase natural circulation flow in a closed loop system undergoing heating/cooling. The buoyancy head is obtained from;

$$\Delta P_{boyancy} = g\ L\ \Delta\rho \tag{1}$$

where $g$ is the gravitation constant and $L$ is the effective elevation difference between the heat source and cooling sink. The density difference, $\Delta\rho$, is proportional to the differential temperature between hot and cold fluid and is given by

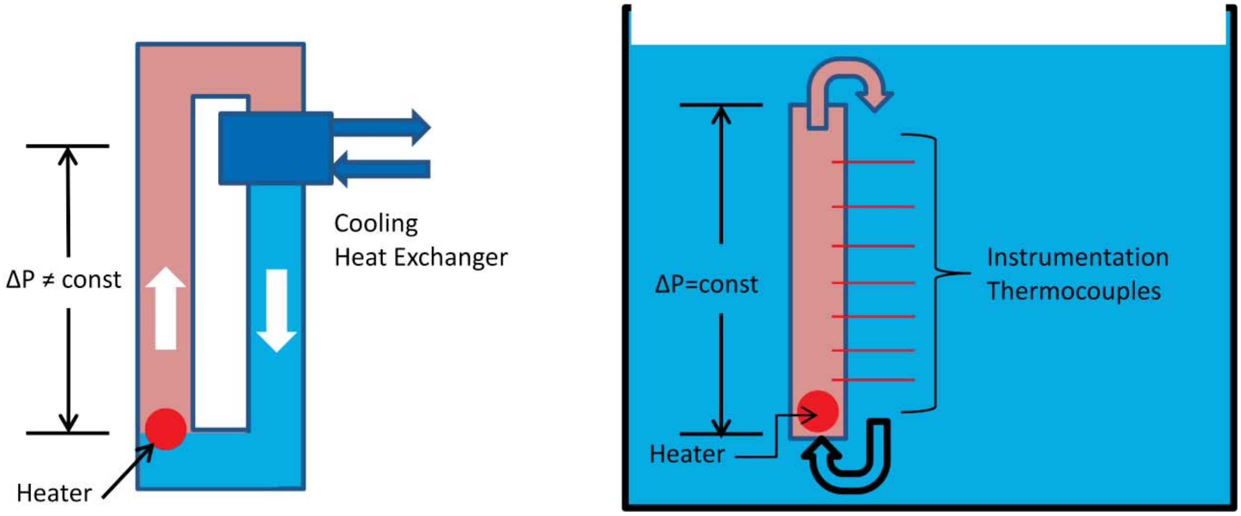


Figure 1. A sketch of the traditional natural circulation loop is shown (left) compared with the idealized setup to impose constant pressure drop boundary condition (right).

$$\Delta\rho = \beta\,\Delta T$$

where $\beta$ is the thermal expansion coefficient. The temperature difference, $\Delta T$, is obtained from the energy balance in the reactor core,

$$\Delta T = \frac{Q}{c_p\,\dot{m}}$$

where $Q$ is the reactor power, which is considered constant in time by neglecting reactivity feedback. The coolant mass flow rate is $\dot{m}$ and $c_p$ is specific heat of the fluid.

The above relationships are combined with Eqn. (1) to give the buoyancy head that is produced in a single-phase natural circulation system by heating and cooling the fluid;

$$\Delta P_{boyancy} = \frac{g\,L\,\beta\,Q}{c_p\,\dot{m}} \tag{2}$$

In addition to this, a relationship for form and friction pressure losses, $\Delta P_{fric}$, is obtained by lumping all flow resistance components into a single coefficient, which is appropriate when the Boussinesq approximation is used where the mass flow rate as function of time is considered uniform around the loop. Thus,

$$\Delta P_{fric} = \xi\,\dot{m}^2 \tag{3}$$

where $\xi$ is the loop friction coefficient. The steady state balance requires

$$\Delta P_{fric} = \Delta P_{boyancy} \tag{4}$$

Substituting Eqn. (2) and Eqn. (3) into Eqn. (4) and rearranging results in

$$\xi = \frac{g\,L\,\beta\,Q}{c_p\,\dot{m}^3} \tag{5}$$

This well-known relationship for closed loop natural circulation flow indicates that the steady state natural circulation flow is proportional to the cubic root of the power.

Next, the dynamic momentum balance can be written as;

$$I\,\frac{d\,\dot{m}}{dt} = \Delta P_{boyancy} - \Delta P_{fric}$$

where $I$ is the loop inertia (defined as the sum of the length-to-area ratio of all the loop components). The buoyancy head under transient conditions is obtained using the average density along the riser, thus

$$\Delta P_{boyancy}\left(t\right) = \frac{g\,L\,\beta\,Q}{c_p \tau_0} \int_{t}^{t-\tau_0} \frac{dt'}{\dot{m}\left(t-t'\right)}$$

where the integral used for the averaging is taken over, $\tau_0$, which is the time period it takes for a change in density resulting from a change in heating to completely fill the riser.

The buoyancy head can be approximated by considering that it is delayed by half the time needed to fill the riser, $\tau$. Thus

$$I\,\frac{d\,\dot{m}\left(t\right)}{dt} = \frac{g\,L\,\beta\,Q}{c_p\,\dot{m}\left(t-\tau\right)} - \xi\,\dot{m}^2\left(t\right) \tag{6}$$

The time constant, $\tau$, is related to the total mass of fluid in the riser, $M$, and mass flow rate as follows

$$\tau = \frac{M}{2\dot{m}} \tag{7}$$

## 3. ANALYTICAL SOLUTION

A small perturbation of the following form can be applied to the dynamic momentum equation in Eqn. (6)

$$\dot{m}\left(t\right) = \dot{m}_0 + \delta\dot{m}\left(t\right)$$

where $\dot{m}_0$ is the initial steady state value of the mass flow rate, and $\delta\dot{m}\left(t\right)$ is a small time-dependent component. The momentum balance in Eqn. (6) is rewritten for small perturbation as,

$$I\,\frac{d\,\delta\dot{m}\left(t\right)}{dt} = \frac{g\,L\,\beta\,Q}{c_p\left(\dot{m}_0 + \delta\dot{m}\left(t-\tau\right)\right)} - \xi\,\left(\dot{m}_0 + \delta\dot{m}\left(t\right)\right)^2$$

Linearizing, by neglecting higher order perturbations yields

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=\frac{g\,L\,\beta Q}{c_p\,\dot{m}_0}\left(1-\frac{\delta\dot{m}(t-\tau)}{\dot{m}_0}\right)-\xi\left(\dot{m}_0^2+2\dot{m}_0\,\delta\dot{m}(t)\right)$$

Simplify using steady state balance,

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=-\frac{g\,L\,\beta Q}{c_p\,\dot{m}_0^2}\,\delta\dot{m}(t-\tau)-2\xi\,\dot{m}_0\,\delta\dot{m}(t) \tag{8}$$

Defining

$$a=\frac{g\,L\,\beta Q}{c_p\,\dot{m}_0^2\,I}=\frac{\xi\dot{m}_0}{I} \tag{9}$$

and substituting Eqn. (5) and Eqn. (9) into the small perturbation dynamic equation, Eqn. (8), the following relationship is obtained;

$$\frac{d\,\delta\dot{m}(t)}{dt}=-a\,\delta\dot{m}(t-\tau)-2a\,\delta\dot{m}(t) \tag{10}$$

The small time-dependent component response is an exponentially damped sinusoidal typical of a harmonic oscillator, thus,

$$\delta\dot{m}(t)\sim e^{(\sigma+i\omega)t} \tag{11}$$

where $\omega$ is the oscillation frequency in radians per second. The real part, $\sigma$, is a negative-valued damping coefficient for stable (decaying) oscillations and positive for growing oscillations. Substitute Eqn.(11) into the linearized momentum balance equation shown in Eqn.(10) and simplifying results in

$$\sigma+i\omega=-a\,e^{-(\sigma+i\omega)\tau}-2a \tag{12}$$

Recognizing the identity

$$e^{-ix}=\cos(x)-i\sin(x)$$

Eqn.(12) can be separated into real and imaginary parts to yield

$$\sigma+2a=-a\,e^{-\sigma\tau}\cos\omega\tau$$
$$\omega=a\,e^{-\sigma\tau}\sin\omega\tau$$

Solve for $\omega$ in terms of $\sigma$ by squaring and adding, and solve for $\sigma$ in terms of $\omega$ by dividing. Therefore,

$$\omega^2+(\sigma+2a)^2=a^2\,e^{-2\sigma\tau}\left(\sin^2\omega\tau+\cos^2\omega\tau\right)=a^2\,e^{-2\sigma\tau}$$
$$\frac{\omega}{\sigma+2a}=-\tan\omega\tau$$

Apply algebraic simplifications to get two transcendental equations for $\sigma$ and $\omega$ ,

$$\omega = \sqrt{a^2 e^{-2\sigma\tau} - (\sigma + 2a)^2} \tag{13}$$

$$\sigma = -2a - \frac{\omega}{\tan \omega\tau} \qquad \text{for} \quad \omega\tau < \pi \tag{14}$$

which are needed to get the decay ratio

$$DR = \exp(2\pi\sigma/\omega). \tag{15}$$

Proof of unconditional stability

The stability boundary corresponds to the condition of $DR = 1$ which is equivalent to $\sigma = 0$. For the case of being on the stability boundary, substituting $\sigma = 0$ in Eqn.(13) yields

$$\omega = \sqrt{a^2 e^{-2\sigma\tau} - (\sigma + 2a)^2} = a\sqrt{-3}$$

which is not a real number. Therefore, for a real frequency it is necessary that $\sigma < 0$ , which leads to the important conclusion that stability is unconditional.

## 4. MODEL EXTENSIONS

The model described above is applicable to the idealized case of single-phase flow with constant pressure drop boundary condition and fixed heat source. Model extensions are presented next to relax some of these assumptions. These include consideration for boiling in the riser, pressure drop balance corrections, time-dependent fission heat source due to reactivity modulation, and heat sink dynamic response.

### 4.1. Extension to Two-Phase Flow due to Bulk Boiling in the Riser

The flow bulk boiling occurs when the flow enters the reactor core in the saturation state. Assuming homogeneous flow as a convenient approximation, the same equations for single phase flow apply but with altered numerical values of the coefficients. Instead of considering the thermal expansion of the liquid as the driver of the buoyancy force, phase change due to the same enthalpy addition affects a decrease of the mixture density. Under the operating pressure of the NuScale module, the density head per unit of enthalpy addition due to bulk boiling is nearly 6 times its value due to liquid expansion. However, for the same reactor power, the steady state mass flow rate will be higher. At the end, the same equation is used where the value of the system parameter, $a$ , is altered. The conclusion of unconditional stability does not depend on the value of the system parameter, and remains in force under the conditions of bulk boiling. There are implied assumptions though, mainly that friction is concentrated at the core inlet and no phase shift adjustment is needed. The assumption of homogeneous flow is also reasonable because vapor-liquid slip would simply reduce the vapor residence time in the riser and reduce the buoyancy force compared with the no slip assumption -- in effect the numerical value of the system parameter would be altered but the equation form remains invariant and the unconditional stability conclusion holds.

## 4.2. EFFECT OF SUBCOOLED BOILING

Unlike the case of bulk boiling where the flow entering the core is saturated, the case of subcooled liquid entering the core and exiting at or near saturation is considered next. There is one important difference between the two cases: for the near-saturated core exit case the steady state mass flow rate is nearly the same as the single phase case. Yet, the effect of added enthalpy on the buoyancy driving head is markedly higher than the liquid expansion case due to phase change. Essentially, the driving head perturbation is increased while the friction term is unchanged compared with the single-phase case. The magnitude of the driving head increase is limited to a factor of 6, where smaller value can be considered due to vapor-liquid slip and the bubble collapse as the bubbles travel along the riser in case of subcooled boiling. The system equation becomes

$$\frac{d\,\delta\dot{m}(t)}{dt} = -\gamma a\ \delta\dot{m}(t-\tau) - 2a\ \delta\dot{m}(t)$$

where the boiling amplification factor, $\gamma$, is a multiplier representing the increase in buoyancy perturbation due to boiling. The value of this multiplier depends on pressure, where $\gamma \approx 6$ for water at the high pressure typical of NuScale operation.

Applying the harmonic solution as was done to the single-phase system, we get the linearized momentum balance equation

$$\sigma + i\omega = -\gamma a\ e^{-(\sigma+i\omega)\tau} - 2a$$

Separate the real and imaginary parts to get

$$\sigma + 2a = -\gamma a\, e^{-\sigma\tau}\cos\omega\tau$$
$$\omega = \gamma a\, e^{-\sigma\tau}\sin\omega\tau$$

Two transcendental equations for $\sigma$ and $\omega$ are obtained. Thus

$$\omega = \sqrt{\gamma^2 a^2\, e^{-2\sigma\tau} - (\sigma + 2a)^2}$$
$$\sigma = -2a - \frac{\omega}{\tan\omega\tau}$$

It can be shown that the condition for instability, i.e. $\sigma > 0$, is realized when $\gamma > 2$ which is possible due to boiling when the core exit conditions are near saturation.

## 4.3. EFFECT OF ARTIFICIALLY BALANCED LOOP

The situation is sometimes encountered in practical applications that the loop balance is not enforced at the initial conditions. For example, a lower flow is imposed to achieve conservatism in calculating thermal limits. In that case, an artificially increased friction, or a pressure drop residual, is imposed. A non-zero pressure drop residual that remains constant during the transient calculation is a common approach. The effect of such residual on the calculated system stability is examined here.

The momentum balance is restored using the residual, $R$, resulting in

$$I\,\frac{d\,\dot{m}(t)}{dt}=\frac{g\,L\,\beta\,Q}{c_p\,\dot{m}(t-\tau)}-\xi\,\dot{m}^2(t)+R$$

Applying a small perturbation to the flow rate using

$$\dot{m}(t)=\dot{m}_0+\delta\dot{m}(t)$$

we get

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=\frac{g\,L\,\beta\,Q}{c_p\left(\dot{m}_0+\delta\dot{m}(t-\tau)\right)}-\xi\left(\dot{m}_0+\delta\dot{m}(t)\right)^2+R$$

Linearizing, by neglecting higher order perturbations, we get

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=\frac{g\,L\,\beta\,Q}{c_p\,\dot{m}_0}\left(1-\frac{\delta\dot{m}(t-\tau)}{\dot{m}_0}\right)-\xi\left(\dot{m}_0^2+2\dot{m}_0\,\delta\dot{m}(t)\right)+R$$

Simplifying using steady state balance, the residual component is eliminated and the differential-difference equation in the flow perturbation is obtained as

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=-\frac{g\,L\,\beta\,Q}{c_p\,\dot{m}_0^2}\,\delta\dot{m}(t-\tau)-2\xi\,\dot{m}_0\,\delta\dot{m}(t)$$

Recall the definition of the system parameter, now defined at the actual power and imposed initial flow,

$$a=\frac{g\,L\,\beta\,Q}{c_p\,\dot{m}_0^2\,I}$$

Calculating the friction coefficient, $\xi$ , using a pair of power and flow values, $\left(Q^*,\,\dot{m}^*\right)$ representing a balanced loop, we define the coefficient

$$b=2\frac{\xi\dot{m}_0}{I}=2\frac{g\,L\,\beta\,Q^*\dot{m}_0}{c_p\,I\,\dot{m}^{*3}}=2a\left(\frac{Q^*\dot{m}_0^3}{Q\dot{m}^{*3}}\right)$$

which reduces the non-zero residual differential-difference equation to

$$\frac{d\,\delta\dot{m}(t)}{dt}=-a\;\delta\dot{m}(t-\tau)-b\;\delta\dot{m}(t)$$

The harmonic mode

$$\delta\dot{m}(t)\sim e^{(\sigma+i\omega)t}$$

is substituted in the linearized momentum balance equation to get for the generally nonzero residual case

$$\sigma+i\omega=-a\;e^{-(\sigma+i\omega)\tau}-b$$

Separate the real and imaginary parts

$$\sigma + b = -a\,e^{-\sigma\tau}\cos\omega\tau$$
$$\omega = a\,e^{-\sigma\tau}\sin\omega\tau$$

Solve for $\omega$ in terms of $\sigma$ by squaring and adding, and solve for $\sigma$ in terms of $\omega$ by dividing. Therefore

$$\omega^2 + (\sigma + b)^2 = a^2\,e^{-2\sigma\tau}\left(\sin^2\omega\tau + \cos^2\omega\tau\right) = a^2\,e^{-2\sigma\tau}$$

$$\frac{\omega}{\sigma + b} = -\tan\omega\tau$$

Apply algebraic simplifications to get two transcendental equations for $\sigma$ and $\omega$ ,

$$\omega = \sqrt{a^2\,e^{-2\sigma\tau} - (\sigma + b)^2}$$
$$\sigma = -b - \frac{\omega}{\tan\omega\tau}$$

Apply the condition of instability threshold, $\sigma = 0$, to calculate $\omega$ we get

$$\omega = \sqrt{a^2\,e^{-2\sigma\tau} - (\sigma + b)^2} = \sqrt{a^2 - b^2}$$

Therefore, instability is achieved when $a > b$ which is equivalent to

$$\dot{m}_0 < \dot{m}^* \left(\frac{Q}{2Q^*}\right)^{1/3}$$

### 4.4. Effect of Power-Reactivity Feedback

The preceding treatment applies to constant power where power-reactivity feedback mechanisms are neglected. It is shown here that power-reactivity feedback is stabilizing and the constant power equations are therefore conservative. In response to a positive flow perturbation, the coolant which also acts as moderator experiences a corresponding increase in density due to temperature decrease. The reduced moderator temperature is associated with an increase in neutron reactivity and a corresponding increase in reactor power. The time lag between a perturbation increasing flow and the resulting response increasing power to the coolant is small given that the time delay (due to flow transit time through the core and the fuel pin heat conduction) is significantly smaller than the dominant riser filling time. Thus, the transient power response can be written as

$$Q(t) = Q_0 + \kappa\,\delta\dot{m}(t)$$

where $\kappa$ is the flow-to-power transfer function, which is considered to be a constant.

Substituting the time-dependent power in the differential-difference equation for the flow perturbation, we must apply the time lag to all time-dependent quantities in the driving head term. Thus,

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=\frac{g\,L\,\beta}{c_p\,\dot{m}_0}\left[Q_0+\kappa\delta\dot{m}(t-\tau)\right]\left[1-\frac{\delta\dot{m}(t-\tau)}{\dot{m}_0}\right]-\xi\left(\dot{m}_0^2+2\dot{m}_0\,\delta\dot{m}(t)\right)$$

Simplify using steady state balance and retaining only the linear terms, we get

$$I\,\frac{d\,\delta\dot{m}(t)}{dt}=-\frac{g\,L\,\beta\,Q_0}{c_p\,\dot{m}_0^2}\,\delta\dot{m}(t-\tau)+\frac{g\,L\,\beta\,\kappa}{c_p\,\dot{m}_0}\,\delta\dot{m}(t-\tau)-2\xi\,\dot{m}_0\,\delta\dot{m}(t)$$

Group variables to define the system parameter, $a$ , as before

$$a=\frac{g\,L\,\beta\,Q_0}{c_p\,\dot{m}_0^2\,I}$$

and define the coefficient

$$a'=a\left(1-\frac{\kappa\dot{m}_0}{Q_0}\right)$$

and substitute into the differential-difference equation to get

$$\frac{d\,\delta\dot{m}(t)}{dt}=-a'\,\delta\dot{m}(t-\tau)-2a\,\delta\dot{m}(t)$$

Comparing the above equation with Eqn.(10) , and noting that $a'<a$ due to the reactivity feedback, it can be concluded that the reactivity feedback has a stabilizing effect.

### 4.5. Effect of Heat Sink Dynamic Response

In the previous analysis it has been assumed that the coolant density in the cold leg is constant implying an ideal heat exchanger. The effect of the idealized heat exchanger assumption is investigated by considering the other extreme, that is a fixed heat sink which allows the density of the coolant in the cold leg to fluctuate with a time lag relative to the density fluctuations in the riser.

The temperature response to flow perturbation arrives at the inlet of the heat exchanger with a time delay relative to heater exit that is twice the time delay from heater exit to the middle of the riser. Assuming the flow areas in the riser and cold leg downcomer are equal, there is an additional time delay of the same magnitude for the temperature perturbation to reach the middle of the downcomer. Thus, the differential-delay of Eqn.(10) can be extended to get

$$\frac{d\,\delta\dot{m}(t)}{dt}=-a\left[\delta\dot{m}(t-\tau)-\delta\dot{m}(t-2\tau)+\delta\dot{m}(t-3\tau)\right]-2a\,\delta\dot{m}(t)$$

which represents the maximum possible feedback from the heat exchanger. A realistic accounting of the heat exchanger dynamics can be imposed using an adjustment factor, $\eta$ , to get

$$\frac{d\,\delta\dot{m}(t)}{dt}=-a\left[\delta\dot{m}(t-\tau)-\eta\left(\delta\dot{m}(t-2\tau)-\delta\dot{m}(t-3\tau)\right)\right]-2a\,\delta\dot{m}(t) \qquad (16)$$

Notice that the ideal heat exchanger equation is recovered by specifying $\eta = 0$ , and the maximum downcomer feedback is obtained for $\eta = 1$. The realistic situation corresponds to a value in the range $0 < \eta < 1$. There is no obvious way in this simple analytical treatment to quantify the heat exchanger adjustment parameter for a particular experiment or a real device such as the NuScale reactor. However, for normal operation near rated conditions for a large device like NuScale reactor with an efficient heat exchanger, it is anticipated that $\eta \approx 0$ is a reasonable approximation, while $\eta$ is somewhat greater than zero at low flow and power where the steam generator heat exchanger is isolated or running at very low feedwater flow rate going through the secondary side in order to avoid cooling the entire system.

It is not easy to find an analytical solution for Eqn.(16) but it can be integrated numerically. This was done for $\eta = 0, 0.1, 0.2, ..., 1$ and the results are shown in Figures (2-a) through (2-e) for $\eta = 0, 0.2, 0.4, 0.6,$ and $0.8$, respectively. It can be verified that for the ideal heat exchanger case, $\eta = 0$, the decay ratio obtained by numerical integration is in agreement with the analytical result. It can be also shown that increasing the adjustment factor, allowing stronger response from the heat exchanger, destabilizes the system as expected. The increase of decay ratio as function of the adjustment factor is shown in Figure 3.

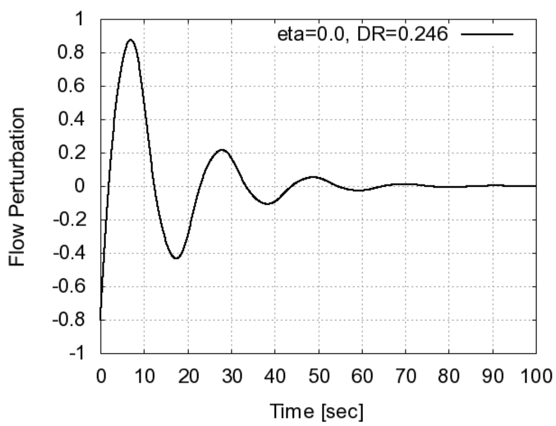


Figure 2-a Calculated flow versus time for $\eta = 0$

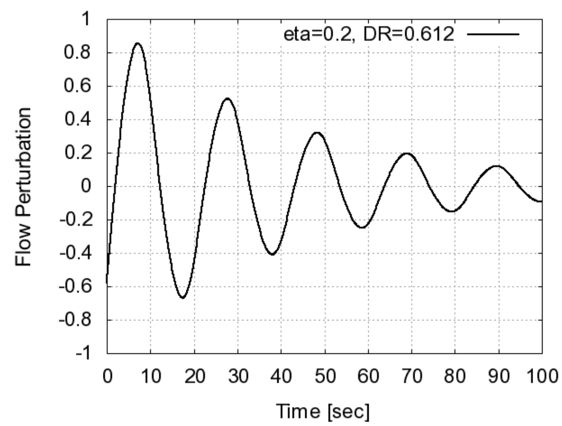


Figure 2-b Calculated flow versus time for $\eta = 0.2$

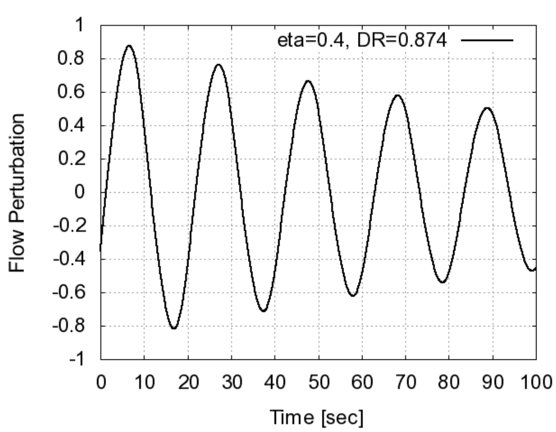


Figure 2-a Calculated flow versus time for $\eta = 0.4$

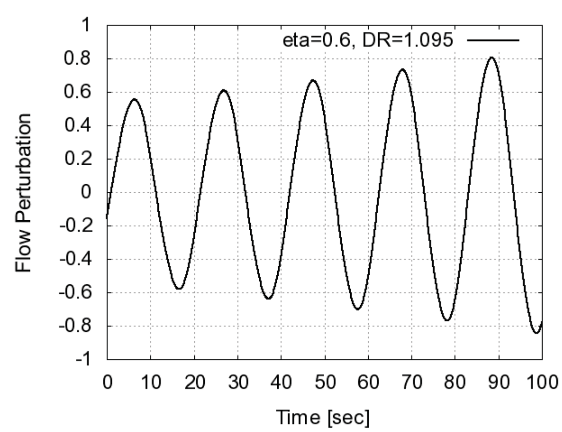


Figure 2-b Calculated flow versus time for $\eta = 0.6$

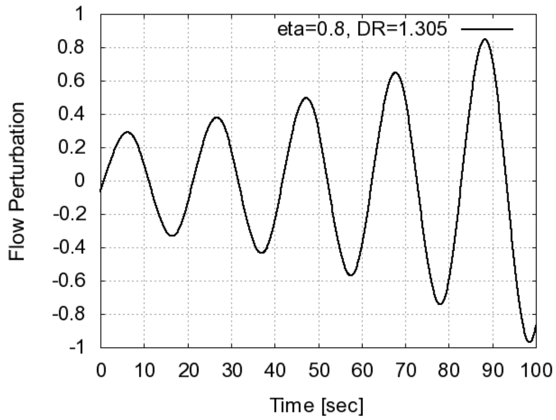


Figure 2-e Calculated flow versus time for $\eta = 0.8$

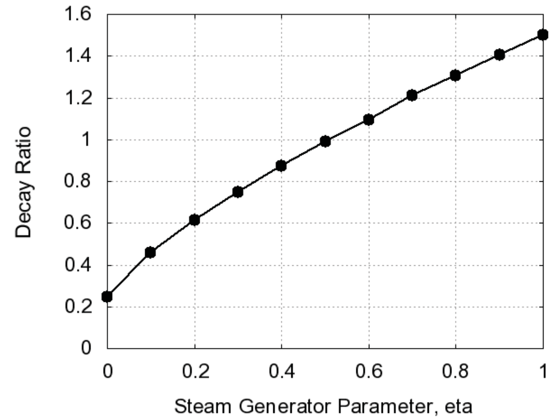


Figure 3 Calculated ecay ratio as function of $\eta$

## 5. CONCLUSIONS

A simple analogue was constructed to examine the stability of the natural circulation flow in NuScale primary loop. It has been shown analytically that the system under consideration is unconditionally stable under single-phase natural circulation flow. As a corollary, the case of bulk boiling where the flow enters the core in saturation is similarly unconditionally stable. As a second corollary, the case of limited boiling where the flow exits the core near saturation can be unstable. Essentially, as the operating conditions are adjusted by increasing power or reducing the heat sink in order to increase the coolant temperature, the system can be destabilized in the narrow range between the conditions of inlet saturation and exit saturation, while the system is unconditionally stable on the two sides of this narrow range. One practical conclusion of this exercise is that the stability of the NuScale module can be assured by avoiding boiling in the riser.

Interesting insights could be obtained from analytical exercises with the stability analogue. It was shown that a flow instability threshold, that is less than the flow calculated by natural circulation in a balanced loop, could be obtained from the analogue by imposing a non-zero pressure drop residual. Another insight is a proof that the power-reactivity feedback has a stabilizing effect.

The analytical solution of the simple analogue equation is free from numerical diffusion and damping effects, and can thus be used to examine and adjust numerical algorithms that are based on finite volume formulation by applying them to the same simple conditions and geometry.

The conclusion of unconditional stability of the natural circulation loop is attached to the assumption of an idealized heat sink that is capable of maintaining a constant temperature and density in the cold leg. This may explain the varying outcome of experiments where the heat exchanger in the cold leg was varied in design. While the heat exchanger used as a steam generator in NuScale reactor can be assumed to perform close to this ideal assumption, it may not accurately apply at very low power and flow consistent with startup. Because of the apparent sensitivity to the design of the heat exchanger, this analytical model can be used without ambiguity to verify stability numerical algorithms and codes in the special case where the cold leg density is fixed.

Experimental verification suggests itself at this stage, where an open loop is used instead of a closed loop. The suggested experimental apparatus is made up of a single tube where a heating element is placed at its bottom. The tube is placed in a large tank to close the flow path and thus the constant pressure drop boundary condition is assured. The tube should be instrumented to measure the flow characteristics at different power levels.  The results should be more robust compared with the closed rectangular tube arrangements with a heater and a heat exchanger and the results can be compared with analytical and numerical studies for algorithmic verification purposes.

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