SIMULATION OF INADVERTENT ROD CLUSTER CONTROL ASSEMBLY BANK WITHDRAWAL AT POWER FOR ANGRA I NPP USING AN LQR DIGITAL CONTROLLER

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ABSTRACT

This paper simulates a rod cluster control assembly bank withdrawal at full power for a PWR incorporating a linear quadratic regulator (LQR) for evaluating the plant stability behavior before actuation of the reactor protection system. The results obtained are quantitatively compared with those presented in the final safety analysis report (FSAR) of Angra I nuclear power plant, a Westinghouse-designed 640 MWe reactor. The results obtained by our simulation model show that power response, pressure, and average coolant temperature to a rod cluster control assembly withdrawal incident starting from full power are in accordance with those presented in FSAR. Higher control flexibility has been observed and also all local controllers were integrated into a unique LQR global controller, which in turn allows easier insertion of restrictions to state variables. All calculations and plots were generated by MATLAB™.

KEYWORDS
Simulation, Reactivity transients, Digital control systems, Stability

1. INTRODUCTION

New nuclear power plant designs are including integrated I&C digital systems for protection, control, alarming and monitoring. Existing operating nuclear power plants, as is the case of Angra 1 nuclear power plant, have to consider the replacement of their I&C analog systems by digital systems for retrofitting their facilities. However, before replacing analog control loops by digital ones it is necessary to design and evaluate their performance, which requires plant and control system modeling with extensive simulations under several normal and abnormal operation conditions.

This paper simulates a rod cluster control assembly bank withdrawal at power for a PWR incorporating a linear quadratic regulator (LQR) for evaluating the plant stability behavior before actuation of the reactor protection system. The results obtained are quantitatively compared with those presented in the final safety analysis report of Angra I nuclear power plant, a Westinghouse designed 640 MWe reactor.

The simulation model consists of the reactor, pressurizer and steam generator as typically used, for example in simulations for the Robinson nuclear power plant [1]. For the stability analysis, state-space variables such as pressurizer pressure, steam generator water drum level, core temperatures (fuel and coolant), steam pressure, power level and reactivity are compared to those of the final safety analysis report (FSAR) in terms of reactor trip to observe the control behavior and performance and thus reduce variable oscillations and keep the plant under operation, and consequently increase its availability and safety performance. The control loop variables are bank reactivity, pressurizer spray flow, pressurizer heater power, steam generator flow and feedwater flow.

This paper is organized as follows. Section 2 displays a literature review on the subject. Section 3 is dedicated to the dynamic model used. Section 4 discusses the case study concerning Angra I nuclear plant. Results and discussions concerning state variables considering and then not considering the LQR digital controller are presented in Section 5. Finally, Section 6 presents the conclusions and recommendations for further research.

2. LITERATURE REVIEW

The vertiginous development of computer technology has spread to practically all fields of human activity. In Nuclear Power Plants (NPP) high performance computer systems have an increasing presence, particularly in the transition from traditional control systems based on relay logics to modern digital control systems. The advantages and new possibilities of digital systems is unquestionable, but they also represent an important challenge not only for the control systems field, but also for others, such as man-machine interface, human factors, NPP safety and safety analysis. In this context, the simulation of NPP dynamic behavior, based on a model that integrate neutronics and thermal-hydraulics with digital controllers, constitutes an important framework for research activities. Two main fields of application are the analysis of NPP response to nuclear and thermal-hydraulic parameter perturbations and also the improvement of operator understanding about control systems behavior through personnel training and computer–aided support systems.

The development of models oriented to NPP dynamic analysis for pressurized water reactors (PWR) can be appreciated through several important contributions published during the last 40 years.

Kerlin et al [1] reported a dynamic model developed for the H.B. Robinson NPP, including the reactor core, pressurizer, primary system piping and steam generator (SG). The model consists of a set of first-order differential equations, comprising point kinetics equations for 6 groups of delayed neutrons; a nodal approximation for reactor fuel and coolant temperatures with one node for fuel and two nodes for coolant; pressurizer balance equations assuming a permanent saturation condition; a simplified model of steam generator represented by 3 regions (primary fluid, tube metal and secondary fluid); two piping sections (hot-leg and cold-leg) and 4 plenums (reactor upper, reactor lower, SG inlet and SG outlet), each considered as a well-mixed volume. Equations numerical coefficients were determined from H.B. Robinson NPP specific data. Concerning the equations linearization for the pressurizer, Kerlin et al [1] cited Thakkar [2] who combined mass, energy and volume balance equations for the derivation of the differential equation for pressurizer pressure, making use of the equation of state (gas law) for steam. A controller was modeled by Kerlin et al [1] only for the pressurizer, where the Sequoyah controller parameters were used. The pressurizer controller uses the normal operating heaters to compensate heat losses and normal pressure variations. The model was implemented using the MATEXP code. Two main types of perturbations were analyzed and experimentally correlated with actual plant behavior. The results of control rod perturbations showed a satisfactory agreement but this was not the case for steam valve perturbations. The authors considered as a probable cause the absence of the SG feedwater control system in the system model.

Nair and Gopal [3] started from the NPP dynamic model developed in [1] and performed the application of optimal control theory in order to simplify the multi-input mathematical control model used in [1], replacing it by a single-input model. Unlike the model used in [1], the pressurizer is assumed to be large enough to accommodate SG primary volume surges and, consequently, the model of Ref. [3] does not need to include the pressurizer dynamics. The NPP is modeled operating at a fixed power level in a load-following mode with a linear controller using as single input the deviation of the steam flow rate from its steady-state value. The controller design considers the existence of various parameters and approximations with estimation errors tending to take the controller response apart from the desired one. The problem is solved introducing in the controller design a sensitivity term, which is minimized. This is done through the system cost weighting matrix Q, properly chosen to give more weight to the system crucial variables in order to minimize the deviations of these variables from the steady-state condition. The reported diagonal matrix Q is composed by element weigh values of 0.0, 0.1 and 1.0, repeated in several positions. The controller feedback gain matrix was obtained with and without sensitivity measures and the model was evaluated for both conditions, considering perturbations on reactor lower plenum temperature. The results show that for the same perturbation a much lower deviation from the steady-state condition is achieved when the controller feedback gain includes sensitivity measures.

Guimarães has presented more advanced SG models applied to Angra NPP. The models were solved using the MATLAB™ package. Firstly, two variants of a teaching purpose model were developed in [4]. The simplest one splits the SG in 3 regions as in [1] (3-variable model). The more detailed one divides the initial 2 regions of the primary side into 4 regions, to separately consider the ascending and descendent primary coolant flux inside the SG U-tubes (5-variable model). No controller model was used. More recently, it has been reported in Ref. [5] a SG model with a nodalization scheme composed by 7 nodes; 2 nodes for the primary region and 2 nodes for the metal tube region (as in the 5 variable model), together with a more detailed representation of the secondary side, which has been divided into 3 regions. The result is a 9-variable model. A special type of PI (pressure indicator) controller was used, with 3 input signals (three-element PI controller): feed water flow, steam flow and water level. The model response was tested with several perturbations, including 10% increase and 10% reduction in the steam valve opening. The model reproduced well the “shrink and swell” phenomena, corroborating the adequacy of the three-element PI controller, adopted for that reason.

Hwang and Burnaby [6] proposed an intelligent control system based on a neural network, able to reduce the effects of uncertainties in models and parameters, to achieve a more stable NPP control in response to perturbations. This intelligent controller was applied to the original model of Ref. [1], which served as a base case. The number of state variables considered for control purposes was reduced from the original amount of 14 in Ref. [1] to 8 state variables used as inputs of the neural network. The network was structured in three layers, with a hidden intermediate layer of 10 nodes and a single output that controls the steam flow rate. The controller design also considered sensitivity analysis for most variable parameters, specifically, moderator and fuel temperature coefficients of reactivity, as suggested in [3]. The network was repeatedly trained, using data generated by a model-based static projective suboptimal controller and a back propagation learning algorithm, to adjust the node weights until the desired overall mean square error, set as 0.003, was achieved. The model was tested with a perturbation of 2 ^{o}F in reactor lower plenum temperature. The test included five scenarios with different combinations of temperature coefficients of reactivity, to evaluate the controller stability to variations in these sensitive parameters. The results were compared with the static projective suboptimal control, showing that the proposed neurocontroller is more effective in robustly achieving stable control under significant reactivity coefficients fluctuations.

The adjustment of controller parameters after NPP uprating is an important application of dynamic models, as can be seen in [7] where the authors evaluated the load rejection transient, from 100% to 5%

of turbine power, which is the bounding case for the actuation of steam dump valves that have to accommodate the increased thermal load without plant trip. The 4.5% power uprating of NPP YGN 1 & 2 and Kori 3 & 4 reduced the capacity of steam dump valves from 70% to 59.1%, increasing the NPP trip frequency derived from large load rejection. They developed an NPP dynamic model, including the control systems related to SG level: feedwater control system and steam dump control system. It was crucial to reduce the fluctuation of SG water level due to swelling and shrinking. They optimized the controller setpoints to cope with the load rejection bounding transient, without the necessity of major equipment modifications. Specifically, sensitivity studies were performed for different values of the Proportional Band (PB – open setpoint of each steam dump valve bank) and the coefficient k_{31} of the output PI controller of feedwater control system (KPFM). It was concluded that PB = 15.0 and KPFM = 0.6 were appropriate, showing stability against fluctuations of SG level and turbine power, and avoiding NPP trips derived from large turbine load rejection. In a more recent publication [8], SG level control of Kori Unit 4 was optimized for the loss of one feedwater pump transient, determining the necessary change in the control system setpoints.

Mahmoud et al [9] developed several strategies of digital controllers design for the linear state-variable model of H.B. Robinson NPP, described in [1], where control inputs (reactivity and electric heater to pressurizer, besides primary water to steam generator and steam generator feedwater) act additively. Four types of controllers were designed and compared, specifically, a discrete linear quadratic regulator (DLQR) and three variants of the sliding mode control (SMC): adaptive SMC, disturbance-estimator SMC and multi-rate output feedback SMC. The DLQR resulted from the linear matrix inequality (LMI)-based formulation of the LQ control. A linear optimal state-feedback control that achieves the minimum quadratic cost was determined, minimizing not the quadratic cost but its upper bound. The SMC starts from the determination of the optimal sliding function. With the optimal sliding function determined what follows is an appropriate discrete-time algorithm to evaluate this function. Three methods were considered for the function evaluation: an adaptive algorithm ensuring that the system always moves toward the sliding surface (adaptive SMC), a modified algorithm more efficient for slow varying disturbances (disturbance-estimator SMC) and the multi-rate output feedback SMC, where output feedback is used for controller design, appropriate when entire states cannot be measured. The controller responses were evaluated for a scenario characterized by an initial deviation in reactor power, steam pressure and hot-leg temperature. The authors concluded that multi-rate output feedback SMC showed the best performance in presence of disturbance because of its ability to better emulate the behavior of state-feedback control.

Abouelsoud et al [10] also considered as a base case the linear state-variable model of H.B. Robinson NPP [1], where the control inputs (reactivity and electric heater to pressurizer, besides primary water to steam generator and steam generator feedwater) act additively. They proposed an approach that uses the solution of the standard linear quadratic regulator as an initial approximation. Then iteratively, the solution is improved by designing a controller that compensates for the violation of the constraints at each iteration, until the optimal solution is achieved. The paper presents the results of the simulation corresponding to deviations in neutron flux, steam pressure and hot-leg temperature, when the developed linear state feedback controller is applied to the system. The authors conclude that the state feedback controller allows NPP to reach steady state values in a very small time after the perturbation, showing the effectiveness of the proposed technique.

Dynamic models required a frequent evaluation of thermodynamic properties for a wide variety of conditions, particularly for water and steam. Hayward [11] demonstrated that the linear secant-modulus equation based on the average bulk modulus over a pressure range can be used to derive several thermodynamic properties of liquids. The equation is both the most accurate and the most convenient two-constant equation available, because of its simplicity. It is appropriate for pressures below 100 MPa. For water, the addition of a quadratic term extends the range of validity up to several hundreds of MPa.

Garland and Hand [12] developed a very important set of simple analytic functions providing the values of thermodynamic properties of water, such as density, specific enthalpy, specific entropy, and specific heat in both liquid and vapor phases, covering the range below one atmosphere to 21.3 MPa and temperatures ranging from 90 ^{o}C to 450 ^{o}C, with an error less than 1-2%, actually less than 1% in much of the cases. The equations are based on the Taylor expansion of the properties as a function of pressure, about the saturation pressure, considering only the first two expansion terms. This approach represents a great advance in comparison with complex transcendental equations and interpolation algorithms because it can be easily implemented in computer codes, saving computational time and memory consumption. It is also very important the fact that these analytic functions can be easily differentiated and integrated. Finally, Garland et al [13] complemented the previous equations to extend the covered range for low pressures and temperatures, down to 0.08 MPa and 90 ^{o}C. These equations were used in this paper to calculate several thermodynamic properties in our dynamic model.

3. DYNAMIC MODEL

The starting point of our model is that of Ref. [1]. The reactor power was modeled using the point kinetics equations taking into account six groups of delayed neutrons and reactivity feedbacks due to fuel Doppler temperature coefficient and coolant temperature coefficients and pressure coefficient (void fraction). The equations were linearized to be cast into a state-space model format. We modeled a normalized reactor power by taking as reference the reactor nominal power. This will become clear when we discuss our results in Section 5.

For the core heat transfer model we also adapted the linearized model used in [1]. As the application is for the Angra I Nuclear Power Plant, we have used design parameters of this reactor. It should be noted here that the differential equation for modeling the pressurizer pressure we have used has an additional term, as compared to that of Ref. [1]: we have added a term to take into account the pressurizer spray effects for pressure control. The spray modeling is similar to that of Ref. [2]. In this sense, we have improved the model of Ref. [1].

The pressurizer equation in our dynamic model has three coefficients multiplying deviations in pressure, volume expansion, and heater power, as in [1]. However, differently from [1][2], the calculation of these coefficients in this paper follows a similar methodology to that described in [4], where a steam generator linear equation system is solved. The volume expansion coefficient is similar to the spray flow coefficient, except by the fact that the spray and in-surge (out-surge) enthalpies are different, making one positive (in / out-surge) and the other one (spray flow) negative (see Ref. [2]).Details on specific enthalpies, densities and specific volumes data have been taken from [12]. The change in mass (density slopes with temperature variation) in the pressurizer has also been modeled as in [1].

Another modification of the core model we implemented is related to the time variation of the rod reactivity fluctuations. We have added a new equation for the reactivity derivative as a function of a fixed rod speed multiplying the hot leg temperature. In this equation we have two terms: the first is related to the fixed rod speed, and the second one is a variable speed imposed by the controller as a control variable. Estimations of these parameters have been made for Angra 1 Nuclear Power Plant. This modeling has been adapted from [14]. This new equation is useful to treat the reactivity derivatives instead of reactivity itself. In the case of control rods, the reason to keep the default speed setting and change it as the controller action is demanded is as follows. If we work with reactivity rather than its derivative the controller will make sudden changes in reactivity, which may be unrealistic in the actual nuclear plant operation, depending on the maximum reactivity that one or more control rod bank can provide each time.

In what concerns the steam generator model, we have adopted the C model of Ref. [15] because it is more realistic than that in [1]. This steam generator model decomposes the primary flow in two paths: ascending and descending paths. It models also the following parts of steam generator: downcomer, separator / riser, drum water, drum steam, and takes into account separately the boiling and subcooling regions of secondary water. The steam generator and reactor models are coupled by only one variable, the steam generator inlet plenum temperature.

It should be stressed here that our model did not consider turbine and condensate (water – steam cycle systems). A second feature that should be mentioned is that although this model is more elaborate by considering, for example, the steam generator modeled in [15] it is still a linear model for controlling purposes (a linear quadratic regulator – LQR). In the linear quadratic regulator (LQR) used in this paper the control variables are: rod control speed, pressurizer spray flow, pressurizer heater power, steam generator steam flow, and feedwater flow. These models were implemented in MATLAB™.

4. CASE STUDY

The accident of uncontrolled rod cluster control assembly bank withdrawal at power is a category II plant condition, which are faults of moderate frequency. These kind of faults do not propagate to cause condition III (infrequent faults) or condition IV (limiting faults) and are not expected to result in fuel rod failures or reactor coolant system overpressurization. They result at worst in reactor trip but the plant is capable of returning back to operation. Therefore, these features allow a better analysis of the state variables and control variables before plant shutdown by the protection system for a transient having a occurrence frequency between normal operation and limiting accidents.

The accident caused by an uncontrolled withdrawal of one or more banks of control rods in a nuclear reactor is part of a set of transients and accidents discussed in the Final Safety Analysis Report (FSAR) for PWRs, such as the Angra-1 (CNAAA-I) nuclear power plant.

In the case of control the rod withdrawal analyzed here, a positive reactivity value equivalent to a ramp of 1 pcm/sec was chosen since high reactivity insertion rates (between 10 pcm/sec and 100 pcm/sec) result in reactor shutdown due to high neutron fluxes, almost immediately after accident start. For a 1 pcm/sec ramp, the transient is slow enough so that we can observe significant changes in pressure and temperature.

In the process of heat extraction by the steam generator, there is a time delay in relation to heat generation of the reactor core, considering the time constants involved. Because of this, there is an increase in temperatures and pressures in the reactor cooling system until the steam pressure in the secondary system reaches the values of relief and safety valve setpoints in steam generators. Primary system temperatures increase and can cause the minimum DNBR loss, which is a design limit in safety analysis.

Automatic reactor shutdown may occur when the plant achieves one of the following limits of the protection system: (a) high-flux neutron or overpower (116%); (b) DNBR limit by OTΔT (overtemperature); (c) OPΔT (overpower); (d) high pressure in the pressurizer (2410 psig) and (e) high water level in the pressurizer. The limits in (a), (b) and (c) also block the withdrawal of control rods.

The CNAAA-I FSAR graphics show that the above limits are reached in approximately 93 seconds with the pressurizer pressure, nuclear power and average temperature of the primary system in approximately 2350 psig, 110% of rated power (Pn) and 600 °F, with deviations of 110 psig, 10% Pn, and 15 °F compared to the values at the beginning of the accident.

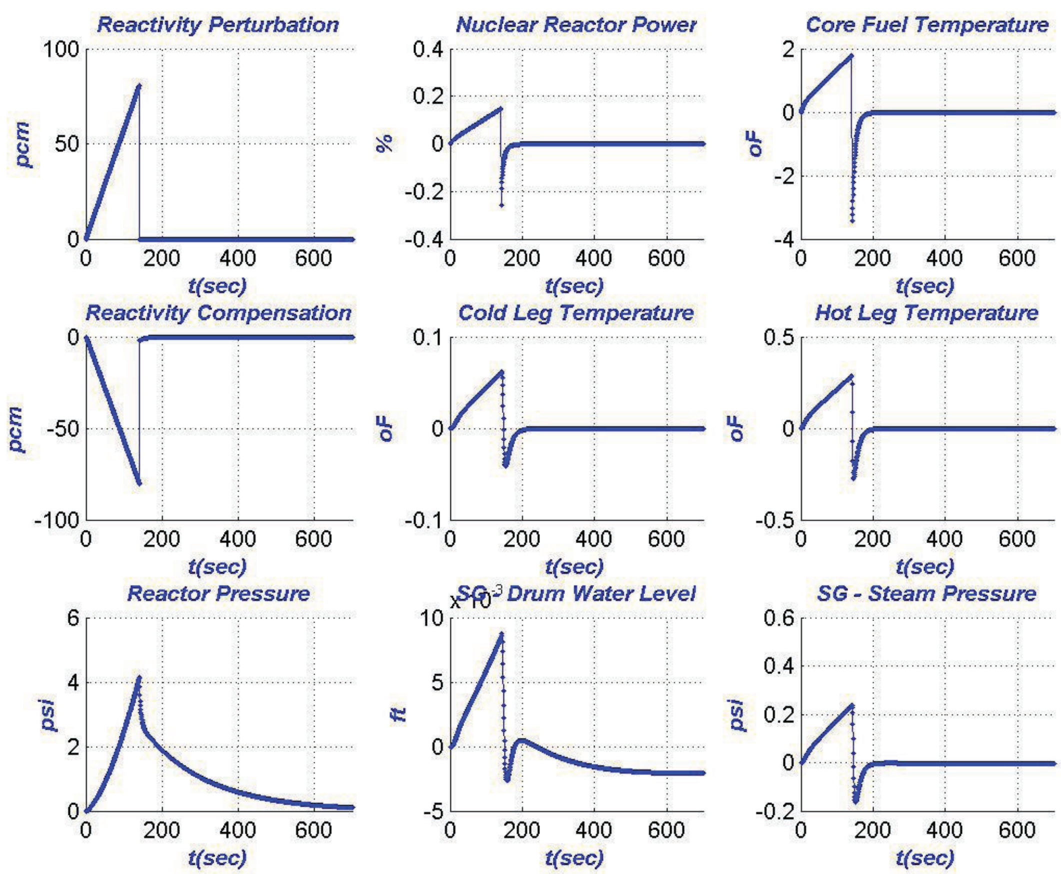
In about 30 seconds, the primary average temperature deviation reaches 2 °F, outside the dead band of the control rod system. There is not, however, actuation of the control rod system, which is the system that shows abnormal behavior. On the other hand, the pressurizer pressure deviation reaches 25 psig within 20 seconds, causing initially an actuation of the pressurizer spray system at 2260 psig. The spray gain is 0.86 lb/sec/psi and the system has at least 1 gpm (gallon per minute) of spray and a maximum flow rate of 400 gpm at 2310 psig. In the opposite direction, we have the proportional heaters working at 2250 psig with a gain of 13.33 kW/psig, with minimal heating of 200 kW at 2235 psig (operating pressure), and a maximum heating of 400 kW at 2220 psig.

For the case of CNAAA-I FSAR, the reactor was shut down when the DNBR reached the value of 1.74 for an average primary system temperature of 600 ° F with minimal reactivity feedback, i.e. moderator reactivity coefficient of 0 pcm/°F and Doppler fuel coefficient –0.91 pcm/^{o}F (least negative value).

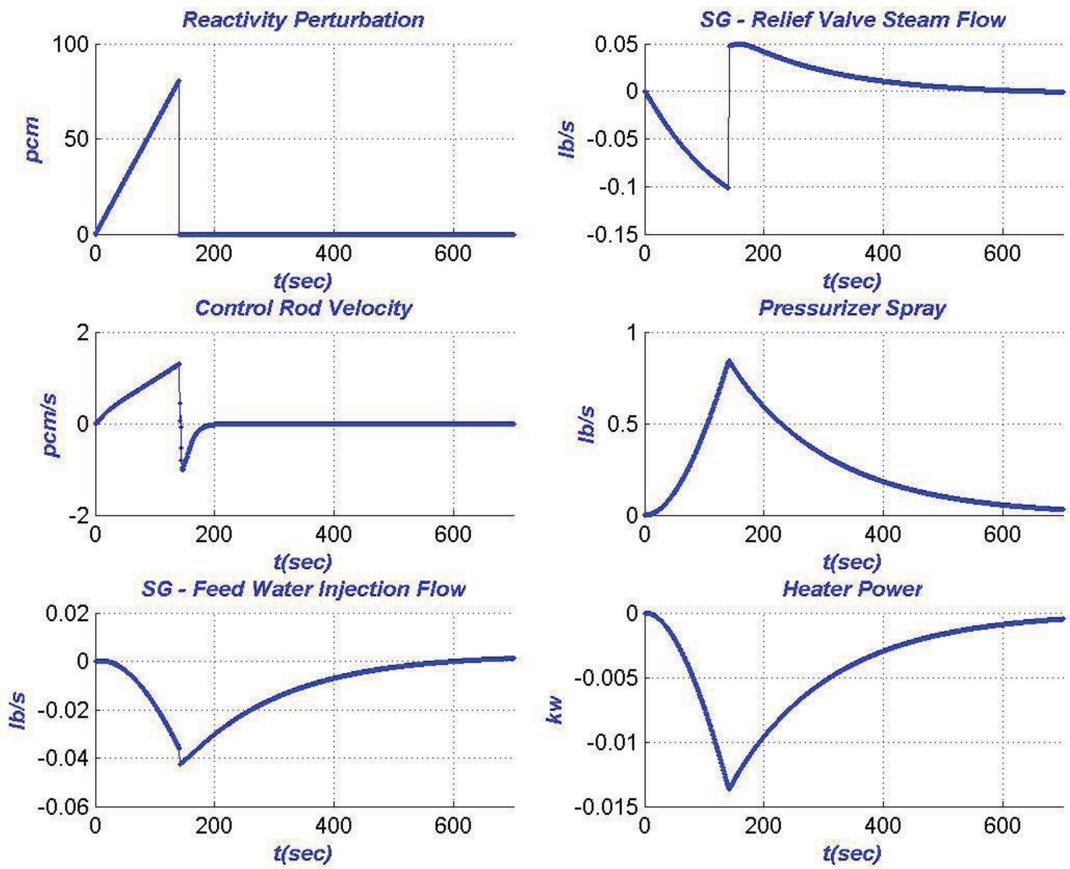
In this paper, we discuss the performance of an LQR digital controller, during the accident of control rod withdrawal at the rate of 1 pcm/sec at CNAAA-I. The dynamic model used was the one discussed in Ref. [1] coupled with the linearized model of steam generators as described in Section 3.

5. RESULTS AND DISCUSSION

The results of the linearized dynamic model described as a state-space model (LQR) are in Figs. 1 – 15.



Figures 1 – 9. State variables behavior. Figures are numbered in columns (first column, Figs. 1 – 3, and so on).



Figures 10 – 15. State variables behavior. Figures are numbered in columns (first column, Figs. 1 – 3, and so on).

Figures 2 – 9 are related to variables representing the primary system behavior and the secondary system concerning the accident of rods uncontrolled withdrawal: reactor power, reactivity of the control rods, fuel temperature, temperatures of the hot leg and cold leg, primary system pressure and secondary steam pressure and water level of the water drum in the steam generator.

Figure 1 shows the positive reactivity insertion caused by the withdrawal of control rods. At 140 sec the disturbance is over, to make it consistent with the accident analyzed in FSAR, where the reactor shutdown occurs at 93 seconds when the increase of the reactor average temperature is such that the DNBR limit by OTΔT (overtemperature deltaT) is reached. In this paper, the average temperature deviation reaches DNBR values at 140 sec without controller.

During 140 seconds, the fuel, coolant, metal (Inconel - steam generator tubes) temperature of the primary and secondary systems, as well as steam pressure and water level in the steam generators showed small or almost negligible deviations. The reactor power peaked at 0.15% Pn. After the accident, the state variables deviations returned to zero with very small oscillations, as shown in Figs. 2 – 9.

Fig. 2 shows that the reactivity was compensated, that is, the controller provided control rod insertion in the same proportion as rod withdrawal, ending insertion after the accident.

Figs. 4 – 9 show that reactor power, fuel temperature, hot leg and cold leg temperatures, steam pressure and drum water level change very little in relation to their initial states, while the primary pressure variation peaked at about 4 psig, as shown in Fig. 3. All variables returned to their initial states shortly after the accident in about 60 seconds in an exponential way with no oscillations. These values are very far from the setpoint values used in FSAR accident analyses for reactor shutdown.

All control variables were demanded by LQR controller, that means, control rod speed provided by the controller in pcm/sec, pressurizer spray and heater power, feedwater flow and steam water in lb/sec. The normal speed of the control rods has been modified by the controller to increase the negative reactivity insertion for a previous speed of 4.64 pcm/sec/°F, a typical value for PWRs. Due to the insertion of negative reactivity by the digital controller the reactor power increased very slowly until 0.15% Pn, very far from the 16% Pn limit. Because of this, there is no significant change in feedwater and steam flows in steam generators.

On the other hand, the heater power and pressurizer spray flow were reduced and increased slightly, respectively, keeping close to the minimum baseline values of 1 gpm and 200 kW, respectively.

A little change in the power of heaters is consistent with the accident scenario, because it does not contribute to the increase of the primary pressure. The spray increased following the pressure tendency.

The behavior of the control variables is shown in Figs. 11 – 15. The plot of control rod speed (Fig. 11) shows that the fixed speed value for control rods in the model, used to insert negative reactivity in the reactor core, was increased by the controller to properly compensate the positive insertion of reactivity caused by the accident of uncontrolled withdrawal of control rods.

The plottings resulting from the implementation of this model in MATLAB™ show that the LQR controller could limit the deviation of the pressurizer pressure at approximately 4 psig, a value that was reached in only 140 seconds. At this instant, the positive reactivity insertion rate was 80 pcm, due to withdrawal of control rods, and the disturbance was terminated.

Regarding the pressurizer pressure, the controller used control variables related to pressure that means, spray and heaters, in the correct direction, i.e., to reduce pressure, which achieves values of relief valves set points in FSAR. The spray was increased by almost 1 pound per second, while the power of the heaters was slightly reduced. With respect to steam generator, the steam flow of relief valves and the feedwater flow are slightly reduced. All control variables returned to zero within about 700 seconds with no oscillations.

For comparison with the FSAR results, we present the plots in Figures 16 – 24, considering the linearized model as state variable equations described above but without actuation of the LQR controller, only with an actuation of a spray flow proportional to pressure variation.

In order to conduct a comparative analysis, the transient was also calculated switching off the LQR controller of the model. The results for state space variables are shown in Figs. 16 – 24. In this case there was no control action on reactivity, heater power, feedwater flow and steam flow. The model considered only one response: the activation of pressurizer spray system, injecting "cold" water to reduce the primary circuit pressure (see Figs. 26 – 30).

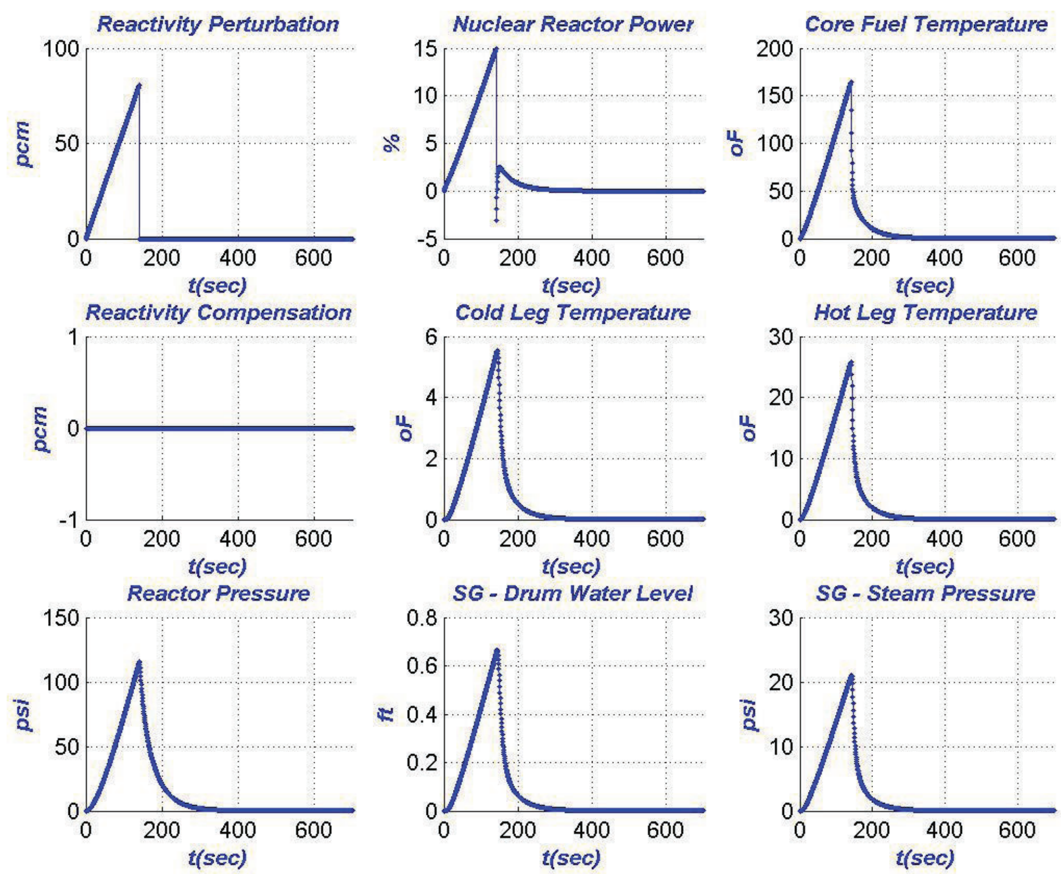
We can observe in variable behavior along time that reactor power, primary system pressure and reactor average temperature follow the same time behavior of variables in FSAR.

Results show that the linearized dynamic model in this paper using the minimum value used in FSAR (-0.91pcm/ºF) causes the reactor average temperature results in 140 sec to be identical to the FSAR value (deviation of 15 ºF) in 93 seconds (reactor trip). The delay of 47 sec can be explained by the differences between the linear model used against the nonlinear model of FSAR.

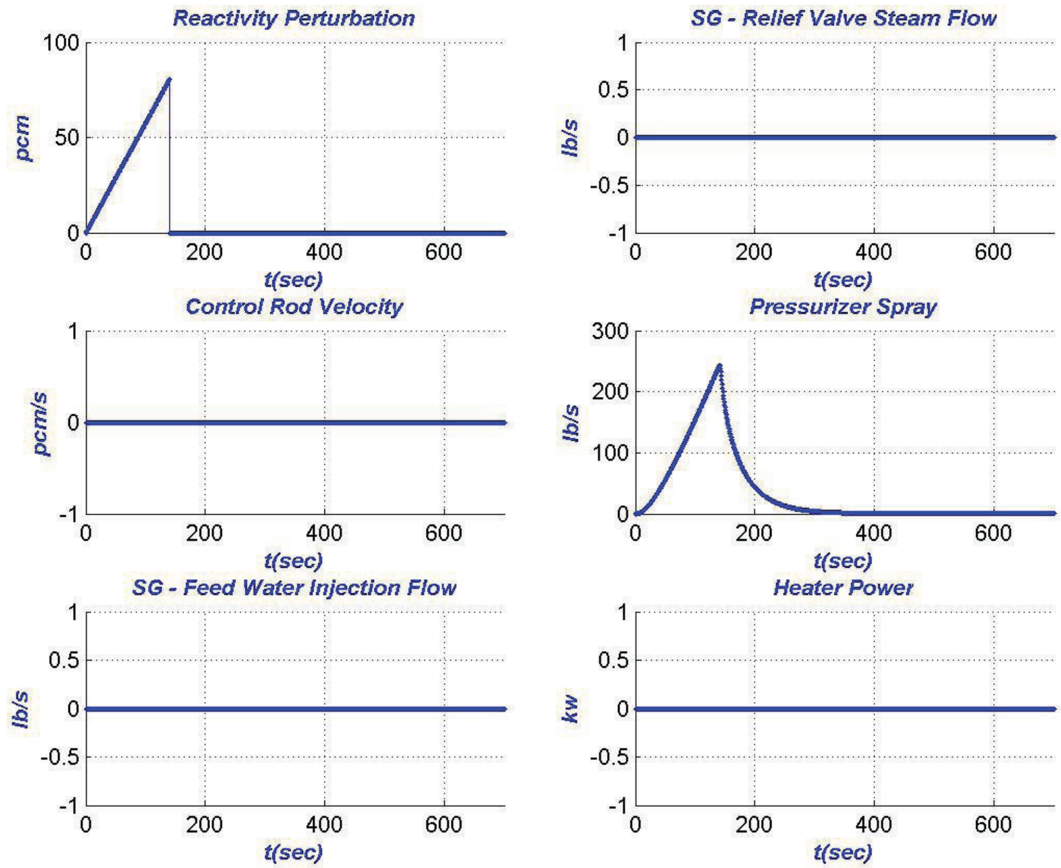
In the case of the pressurizer pressure, the value is lower than that in FSAR at 93 sec. However, it reaches a 115 psig deviation in 140 sec in contrast to the deviation of FSAR (100 psig in 93 sec). In this case, the pressure deviation is increased because there is no actuation of the pressurizer relief valves in our linearized model. The power deviation shows the same behavior (10% deviation in about 100 seconds) as in FSAR.

As previously established, the behavior of state variables during the transient is consistent with FSAR results. Plant performance was considerably improved due to LQR controller effective actuations, characterized by much smaller deviations of state variables.

Actually, the additional negative reactivity feedback, together with the other control variables, resulting from the LQR controller actuation, allowed controlling the transient in such a way that, once the initial perturbation was terminated, the plant finally reached a new steady state condition, without the necessity of an automatic reactor shutdown.



Figures 16 – 24. Reactivity perturbation and some important state variables during the analyzed transient without local controllers (figures are numbered in columns, as Figs. 16-18 are on the first column, and so on).



Figures 25 – 30. Control variables behavior without LQR controller (Figure numbering is in columns (Figs. 25 – 27 are on the left column and Figs. 28 – 30 are on the right column)

Table I shows the results of the comparative analysis done for some main state variables, considering the maximum deviation achieved with and without the LQR controller.

Table I. Comparative analysis of state variables behavior with (Figs. 2 – 9) and without (Figs. 17 – 24) local controllers

State variable	**Maximum Deviation**		
	Without LQR controller	With LQR controller	Relative reduction using LQR controller
Nuclear Reactor Power (%)	15.0	0.15	99.0
Fuel temperature (^{o}F)	170	1.8	98.9
Hot leg temperature (^{o}F)	26	0.25	99.0
Reactor pressure (psi)	115	4	96.5
SG - Steam pressure (psi)	21	0.23	99.9

As can be seen, the LQR controller significantly contributed to reduce the amplitude of state variables deviation, a reduction above 96% in all cases.

The stabilization time of the transient without LQR controller is approximately 400 seconds for all state variables, whereas with LQR controller some variables, like reactor power, core fuel temperature and hot / cold leg temperature stabilize very rapidly, in about 200 seconds. Due to the heat transfer delay, the steam generator drum water level has a greater stabilization time, between 400 and 500 seconds, but comparable to the stabilization time without the LQR controller. Only the reactor pressure kept the same control variables behavior that means a stabilization time of 700 sec.

6. CONCLUSIONS

This paper described a linearized model of a nuclear power plant including reactor core, primary loop, and pressurizer together with spray system and steam generator. This linearized model was cast into a space – state variable format to allow the evaluation of a LQR controller performance. This combined model reactor – controller was applied to Angra I NPP, a PWR Westinghouse reactor simulating an uncontrolled rod cluster control assembly bank withdrawal accident.

The results without controller showed good agreement with FSAR results, although with a delay in DNBR conditions achievement (by high reactor average temperatures) of 47 sec due to the difference between FSAR modelling (non-linear model) and space – state models (linear). On the other hand, the simulation with LQR controller actuation demonstrated the controller efficiency to stabilize the state – space variables before reactor shutdown.

Future developments will include a model expansion to insert all secondary systems (turbine – condensate system and water – steam cycle systems) and to replace the LQR controller by others, like H∞, LQG, non-linear controllers) to evaluate and compare the performance of each one in other accident categories analyzed in FSAR.

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