RAYLEIGH-TAYLOR INSTABILITY OF VISCOUS FLOW WITH HEAT ANS MASS TRANSFER

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ABSTRACT

Film boiling on a horizontal surface is a typical example of the Rayleigh-Taylor instability. During the film boiling, phase changes take place at the interface, so that the effect of phase change on the interfacial instability must be taken into consideration. In addition, the vapor layer is not quite thick that a viscous flow must be analyzed. In this study, we obtain the dispersion relation for the Rayleigh-Taylor instability of viscous fluids with heat and mass transfer. The dispersion relation is numerically solved to investigate the effect of phase change on the growth rate. In addition, critical wavenumber and most unstable wavelength are examined.

1. INTRODUCTION

Film boiling on a horizontal surface is a typical example of the Rayleigh-Taylor instability. During film boiling, the phase change plays an important role in the dynamics of the interface. In addition, since the minimum values of vapor films are of the order of 10~100 micro-meters, the viscous effect can be dominant over the inertia effect (low Reynolds number). However, the Rayleigh-Taylor instability have been usually analysed for isothermal and inviscid flows with an aim to develop boiling heat transfer models such as minimum film boiling heat flux [1-3] and critical heat flux [1,4]. Obviously, one needs to consider the effects of phase change and fluid viscosities.

In terms of viscosity and phase change, the classical Rayleigh-Taylor instability refers to the case of two inviscid fluids without phase changes. [5-6] considered the Rayleigh-Taylor instability for viscous fluids. [7] incorporated the effect of phase change into the Rayleigh-Taylor instability of inviscid fluids. When the fluids are heated from below, the phase change was shown to have the stabilizing effect reducing the growth rate, but it did not change the critical wavenumber. [8] considered viscous fluids with phase change, and showed that viscosity alone or phase change alone has no effect on the critical wavenumber. There was a strong stabilizing effect arising from the coupling of viscosity and phase change. However, his analysis was for two fluids with equal kinematic viscosities. [9] formulated a different form of the dispersion relation that reduces immediately to the form derived by [5] for isothermal flows. Their analysis is easier and more straightforward than the one presented in [8], but their dispersion relation was for semi-infinite two fluids with equal kinematic viscosities. Recently, [10] considered viscous potential fluids with phase change. The properties of two fluids were different. However, the viscous potential approach is inadequate for thin vapour films [11].

The purpose of this study is to obtain the dispersion relation of the Rayleigh-Taylor instability for viscous fluids with heat and mass transfer. The critical and most unstable conditions are investigated.
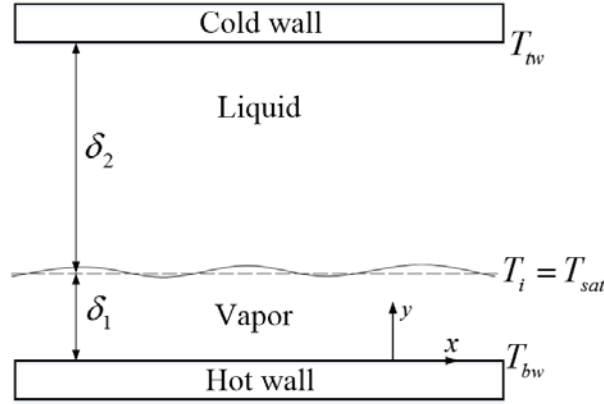

Figure 1 Rayleigh-Taylor instability relevant to nearly saturated film boiling on a horizontal surface

2. RAYLEIGH-TAYLOR INSTABILITY

Figure 1 shows the Rayleigh-Taylor instability relevant to nearly saturated film boiling on a horizontal surface. The vapour is heated from below, and the liquid is cooled from above. Subscripts 1 and 2 indicate the lower fluid (vapour) and the upper fluid (liquid), respectively. The vapour is initially in equilibrium with the liquid at the interface. The liquid layer is thick enough to be nearly semi-infinite in view of perturbation. However, Rayleigh-Benard convection does not set in, owing to the small temperature gradient across the liquid layer (nearly saturated film boiling). In general, the vapour film is not thick. Consequently, the base flow is motionless; and all of the heat conducted across the vapour layer go into the liquid without phase change. Two fluids are assumed to be incompressible.

The linearized equations for perturbations are as follows:

$$\nabla \cdot \mathbf{v} = 0 , \tag{1}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \mu \nabla^2 \mathbf{v} , \tag{2}$$

where $\mathbf{v}$, ρ, μ, and p are the velocity vector, density, viscosity, and pressure, respectively. The interface is perturbed as $\eta = \tilde{\eta} \exp(ikx + \omega t)$, where $\tilde{\eta}$, k, ω, and i are the amplitude of the perturbed interface, the wavenumber, the growth, and imaginary number, respectively. If the horizontal and vertical velocities and perturbed pressure are $u = \tilde{u}(y)\exp(ikx + \omega t)$, $v = \tilde{v}(y)\exp(ikx + \omega t)$, and $p = \tilde{p}(y)\exp(ikx + \omega t)$, one obtains differential equations for the amplitude of the vertical component of the perturbed velocity:

$$\frac{d^4 \tilde{v}_1}{dy^4} - \left(2k^2 + \frac{\omega}{\nu_1} \right) \frac{d^2 \tilde{v}_1}{dy^4} + k^2 \left(k^2 + \frac{\omega}{\nu_1} \right) \tilde{v}_1 = 0 , \quad (0 \le y \le \delta_1) \tag{3}$$

$$\frac{d^4 \tilde{v}_2}{dy^4} - \left(2k^2 + \frac{\omega}{\nu_2} \right) \frac{d^2 \tilde{v}_2}{dy^4} + k^2 \left(k^2 + \frac{\omega}{\nu_2} \right) \tilde{v}_2 = 0 , \quad (y > \delta_1) \tag{4}$$

where ν is the kinematic viscosity.

The no-slip is enforced at the bottom wall (at $y = 0$):

$$\tilde{v}_1 = 0 \text{ and } \frac{d\tilde{v}_1}{dy} = 0 . \tag{5}$$

The liquid is treated nearly semi-infinite in view of perturbation. Hence, for large y,

$$\tilde{v}_2 \to 0 . \tag{6}$$

A phage change is a non-equilibrium process. When a phase change takes place, the vapour temperature is higher or lower than the saturation one at the interface. A general model for phase change (e.g. Hertz-Knusen relation) takes into account discontinuity of temperature at the interface. The temperature jump is however neglected unless phase changes occur too rapidly[12]. Hence, the temperatures of liquid and vapour at the interface are set to the saturation temperature which is determined from the saturation pressure. The variation of the saturation pressure along the interface induces the variation of the interfacial temperature leading to a thermo-capillary effect. However, the large latent heat of phase change makes the interface almost isothermal, and thus the interface temperature variation is too small to induce a significant thermo-capillary force [13]. Therefore, we ignore the thermo-capillary effect. Below are the interfacial conditions at $y = \delta_1$:

$$\tilde{\eta} = \frac{\rho_1 \tilde{v}_1}{b + \omega\rho_1} = \frac{\rho_2 \tilde{v}_2}{b + \omega\rho_2} , \tag{7}$$

$$\frac{d\tilde{v}_1}{dy} = \frac{d\tilde{v}_2}{dy} , \tag{8}$$

$$\frac{\mu_1}{k^2}\frac{d^3\tilde{v}_1}{dy^3} - \left(3\mu_1 + \frac{\omega\rho_1}{k^2}\right)\frac{d\tilde{v}_1}{dy} = \frac{\mu_2}{k^2}\frac{d^3\tilde{v}_2}{dy^3} - \left(3\mu_2 + \frac{\omega\rho_2}{k^2}\right)\frac{d\tilde{v}_2}{dy} + (\sigma k^2 - \Delta\rho g)\frac{\rho_2\tilde{v}_2}{b + \omega\rho_2} , \tag{9}$$

$$\mu_1\left(\frac{d^2\tilde{v}_1}{dy^2} + k^2\tilde{v}_1\right) = \mu_2\left(\frac{d^2\tilde{v}_2}{dy^2} + k^2\tilde{v}_2\right) . \tag{10}$$

Equations (7)~(10), respectively, result from the mass/energy transfer matching, tangential velocity matching, normal stress balance, and shear stress matching at the interface.

The interfacial conditions through Eq. (8)~(10) are readily obtained. We now discuss the interfacial condition of Eq. (7) associated with mass/energy transfer at the interface. This condition was first introduced by [7], conjecturing that the interface temperature is maintained at the saturation temperature corresponding to the base state. To be precise, the interface temperature varies along the interface, but as previously stated, the interface temperature remains nearly constant due to the large latent heat of phase change. Assuming the motionless base state with $K_1(T_{bw} - T_{sat})/\delta_1 = K_2(T_{sat} - T_{tw})/\delta_2$ and neglecting the viscous dissipation, he obtained a linearized relationship between the mass flux and perturbation:

$$\rho_1\left(\mathbf{v}_1 - \frac{\partial\eta}{\partial t}\right)\cdot\mathbf{n} = \frac{1}{L}\left\{\frac{K_2(T_{sat} - T_{tw})}{\delta_2 - \eta} - \frac{K_1(T_{bw} - T_{sat})}{\delta_1 + \eta}\right\} \approx b\eta , \tag{11}$$

$$b \equiv \frac{1}{L}\left[\frac{K_1(T_{bw} - T_{sat})}{\delta_1^2} + \frac{K_2(T_{sat} - T_{tw})}{\delta_2^2}\right] . \tag{12}$$

where $\mathbf{n}$ is the normal interface vector, L latent heat, and K thermal conductivity. The mass flux is the product of a perturbation η and a proportionality constant b. The constant b is a function of the base state properties, and it will be used to measure the degree of phase change. Equation (7) is obtained from conservation of mass: $\rho_1(\mathbf{v}_1 - \partial\eta/\partial t)\cdot\mathbf{n} = \rho_2(\mathbf{v}_2 - \partial\eta/\partial t)\cdot\mathbf{n} = b\eta$. The mass/energy condition of Eq. (7) was often used in [7-10]. More comprehensive formulations for phase changes can be found in studies regarding the onset of instability in the fluid system where a liquid layer underlies a vapour layer [13]. The main difference between existing models is in the predicted magnitude of phase change. All models are able to predict phenomena that evaporation takes place when the interface approaches the

hotter wall, and condensation occurs when the interface approach the colder wall. In this study, we adopted Hsieh's approach in order to simplify the problem, yielding an analytical dispersion relation.

For a non-dimensional analysis, we define $m = \mu_2 / \mu_1$, $r = \rho_2 / \rho_1$, $\alpha = \delta_1 k$, $c = \omega \mu_1 / (\Delta \rho g \delta_1)$, $F = \rho_1 \Delta \rho g \delta_1^3 / \mu_1^2$, $B = \Delta \rho g \delta_1^2 / \sigma$, and $\beta = b \delta_1^2 / \mu_1$. The wavenumber and growth rate are denoted by α and c. The length and the amplitude of velocity perturbation are scaled by δ_1 and $\Delta \rho g \delta_1^2 / \mu_1$, respectively: $\overline{v} = \tilde{v} / (\Delta \rho g \delta_1^2 / \mu_1)$ and $\overline{y} = y / \delta_1$. Now, we write Eqs. (3) and (4) in the non-dimensional forms:

$$\frac{d^4 \overline{v}_1}{dy^4} - (\alpha^2 + p^2) \frac{d^2 \overline{v}_1}{dy^2} + \alpha^2 p^2 \overline{v}_1 = 0, \ (0 \le \overline{y} \le 1) \tag{13}$$

$$\frac{d^4 \overline{v}_2}{dy^4} - (\alpha^2 + q^2) \frac{d^2 \overline{v}_1}{dy^2} + \alpha^2 q^2 \overline{v}_1 = 0, \ (\overline{y} > 1) \tag{14}$$

where $p^2 = \alpha^2 + Fc$ and $q^2 = \alpha^2 + rFc / m$. The boundary/interfacial conditions of Eqs. (5)~(10) are written as

$$\overline{v}_1 = 0 \ \text{and} \ \frac{d\overline{v}_1}{dy} = 0, \ (\text{at} \ \overline{y} = 0) \tag{15}$$

$$\overline{v}_2 \to 0, \ (\text{as} \ \overline{y} \ \text{increases}) \tag{16}$$

$$\overline{v}_1 - \frac{r\beta + rFc}{\beta + rFc} \overline{v}_2 = 0, \tag{17}$$

$$\frac{d\overline{v}_1}{dy} = \frac{d\overline{v}_2}{dy}, \tag{18}$$

$$\frac{d^3 \overline{v}_1}{dy^3} - (2\alpha^2 + p^2) \frac{d\overline{v}_1}{dy} - m \frac{d^3 \overline{v}_2}{dy^3} + (2m\alpha^2 + mq^2) \frac{d\overline{v}_2}{dy} + \alpha^2 (1 - B^{-1}\alpha^2) \frac{rF}{\beta + rFc} \overline{v}_2 = 0, \tag{19}$$

$$\frac{d^2 \overline{v}_1}{dy^2} + \alpha^2 \overline{v}_1 - m\left(\frac{d^2 \overline{v}_2}{dy^2} + \alpha^2 \overline{v}_2 \right) = 0, \tag{20}$$

respectively. Equations (17)~(20) are satisfied at $\overline{y} = 1$. The general solution of Eq. (13) is given by

$$\overline{v}_1 = A_{11} e^{-\alpha(\overline{y}-1)} + A_{12} e^{\alpha(\overline{y}-1)} + A_{13} e^{-p(\overline{y}-1)} + A_{14} e^{p(\overline{y}-1)}. \tag{21}$$

Considering the boundary condition of Eq. (16), the general solution of Eq. (14) is given by

$$\overline{v}_2 = A_{21} e^{-\alpha(\overline{y}-1)} + A_{23} e^{-q(\overline{y}-1)}. \tag{22}$$

The exponent form is intended for easy applications of the interfacial conditions at $\overline{y} = 1$.

Upon substitution of Eqs. (15) and (17)~(20) into Eqs. (21) and (22), we have a linear matrix system for A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, and A_{23}:

$$\begin{vmatrix} e^\alpha & e^{-\alpha} & e^p & e^{-p} & 0 & 0 \\ \alpha e^\alpha & -\alpha e^{-\alpha} & pe^p & -pe^{-p} & 0 & 0 \\ 1 & 1 & 1 & 1 & -R & -R \\ \alpha & -\alpha & p & -p & -\alpha & -q \\ 2\alpha^2 & 2\alpha^2 & \alpha^2 + p^2 & \alpha^2 + p^2 & -2m\alpha^2 & -m(\alpha^2 + q^2) \\ -(\alpha^2 + p^2) & \alpha^2 + p^2 & -2p\alpha & 2p\alpha & m(\alpha^2 + q^2) - JT & 2\alpha qm - JT \end{vmatrix} = 0, \tag{23}$$

where $R = (r\beta + rFc) / (\beta + rFc)$, $J = rFc / (\beta + rFc)$, and $T = \alpha(1 - B^{-1}\alpha^2) / c$.

For non-trivial solutions, the determinant of the matrix must be zero. After considerable algebraic manipulation, we obtained the following dispersion relation:

$$D = A_1 B_1 + (AB)_2 + A_3 B_3 + A_4 B_4 + A_5 B_5 + A_6 = 0, \tag{24}$$

where

$A_1 = R(\alpha^2 - p^2)[JT - mq(\alpha + q)],$

$(AB)_2 = Rm(\alpha + q)[JT - mq^2 - m\alpha^2]B_2$
$\qquad + Rm\alpha(q - \alpha)\{[(R+1)\alpha + m(q-\alpha)]2p\alpha + [2\alpha + m(q-\alpha)]pB_5 + [(R+1)\alpha + m(q-\alpha)]\alpha B_4 - (\alpha + q)mB_2\},$

$A_3 = R^2 m(\alpha^2 - p^2)p(\alpha + q),$

$A_4 = R^2(\alpha^2 + p^2)^2 + R(R+1)m\alpha p^2(q - \alpha),$

$A_5 = 2R^2\alpha^2 p[m(q-\alpha) + 2\alpha],$

$A_6 = \alpha p(\alpha^2 + p^2)[R(R+1)m(q-\alpha) + 4R^2\alpha],$

$B_1 = \alpha \cosh\alpha \sinh p - p \sinh\alpha \cosh p,$

$B_2 = 2\alpha p(1 - \cosh\alpha \cosh p) + (\alpha^2 + p^2)\sinh\alpha \sinh p,$

$B_3 = p \cosh\alpha \sinh p - \alpha \sinh\alpha \cosh p,$

$B_4 = \alpha \sinh\alpha \sinh p - p \cosh\alpha \cosh p,$

$B_5 = p \sinh\alpha \sinh p - \alpha \cosh\alpha \cosh p.$

Equation (24) relate the wavenumber α to the growth rate c. For isothermal flow ($\beta = 0$, $R = J = 1$), the dispersion relation reduces to the form of [6]. Physically, the wavenumber is real and positive, whereas the growth rate may be complex.

3. DISCUSSION

The dispersion relation of Eq. (24) was analyzed with water-vapor properties at the saturation pressure of 150 bar. Recall that b given by Eq. (12) measures the magnitude of phase change. Therefore, $\beta = b\delta_1^2 / \mu_1$ will be used to measure the degree of phase change in the non-dimensional analysis. As the degree of phase change increases, β, R, and J increase as long as the interface is unstable ($c > 0$). Figure 2 (left) shows the growth rate c depending on the wavenumber α when flow is isothermal. F represents the degree of vapor film thickness. F varies from 1 to 10^8. Figure 2 (right) shows that the phase change decreases the growth rate. The arrows indicate the direction of increasing R, i.e., degree of phase change.
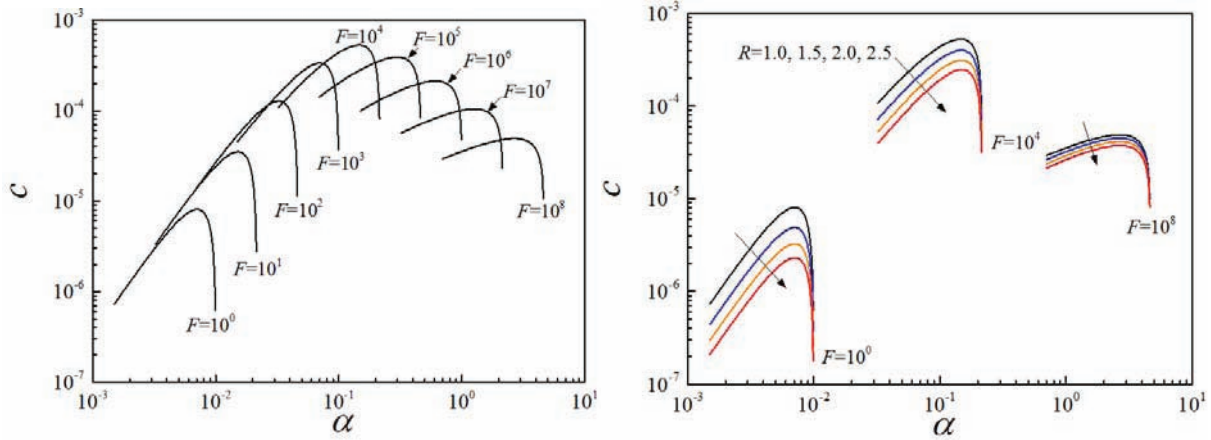
Figure 2. Growth rate as a function of wavenumber. Left: isothermal flow ($R=1$), Right: The arrows indicate the direction of increasing R ($R>1$)

The critical condition corresponds to the point at which the growth rate is zero. For wavenumbers lower than the critical wavenumber, the system is unstable, otherwise, stable. Since Eq. (24) becomes naturally zero when c is zero, the critical equation cannot be obtained by putting $c=0$ into Eq. (24). The critical equation is asymptotically obtained. For an infinitesimal value of c, Eq. (24) is expressed using the Taylor series expansion with regard to c.

$$\lim_{c \to 0}\left[D + \frac{\partial D}{\partial c}c + \frac{\partial^2 D}{\partial c^2}\frac{c^2}{2} + O(c^3) \right] = 0 . \tag{25}$$

Since D and $\partial D / \partial c$ are zero at $c=0$, for Eq. (25) to be satisfied, $\partial^2 D / \partial c^2$ must be zero at $c=0$. As result, we obtain the following critical equation:

$$\beta = (1 - B^{-1}\alpha^2)\frac{rF}{2\alpha}\left[\frac{m(\cosh 2\alpha - 2\alpha^2 - 1) + (\sinh 2\alpha - 2\alpha)}{m^2(\cosh 2\alpha - 2\alpha^2 - 1) + m(\sinh 2\alpha - 2\alpha) + mr\sinh 2\alpha + 2\alpha^2 r + 2\alpha mr + r\cosh 2\alpha + r} \right] . \tag{26}$$

Since $\cosh 2\alpha - 2\alpha^2 - 1 > 0$ and $\sinh 2\alpha - 2\alpha > 0$ for $\alpha > 0$, the square bracket is always positive. Therefore, the RHS(right-hand-side) of Eq. (26) is positive for $0 < \alpha < B^{1/2}$, while it approaches zero as $\alpha \to 0$ or $\alpha \to B^{1/2}$. For isothermal flows ($\beta = 0$), the above equation is satisfied when $\alpha = B^{1/2}$ (or $k = \Delta\rho g / \sigma$). When β is small, there are two values of α such that RHS is equal to β. The largest one is of physical importance. When β is higher than the maximum value of RHS, there is no solution of Eq. (26) for α. In this case, the interface is stable for all wavenumbers.

For a thin layer of vapor ($\alpha = \delta_1 k << 1$), the RHS of (26) is approximated further.

$$\beta = (1 - B^{-1}\alpha^2)\frac{1}{3}F\alpha^2 . \tag{27}$$

In a dimensional form,

$$k^4 - \frac{\Delta\rho g}{\sigma}k^2 + \frac{3\nu_1 b}{\sigma\delta_1^3} = 0 . \tag{28}$$

There is no liquid viscosity term in Eq. (28), which is attributed to the fact that as the vapor layer is thin to the extent of a creeping flow, the vapor viscosity becomes dominant. On the other limit of a thick vapor layer ($\alpha >> 1$), the hyperbolic functions are dominant over polynomial functions. In this case, Eq. (26) is approximated as

$$\beta = (1 - B^{-1}\alpha^2)\left[\frac{rF}{2\alpha}\frac{m\cosh 2\alpha + \sinh 2\alpha}{m^2\cosh 2\alpha + m\sinh 2\alpha + mr\sinh 2\alpha + r\cosh 2\alpha}\right]. \tag{29}$$

Arranging this,

$$2\alpha\left(m + r\frac{m\tanh 2\alpha + 1}{m + \tanh 2\alpha}\right)\beta = (1 - B^{-1}\alpha^2)rF. \tag{30}$$

Since $\tanh 2\alpha \to 1$ as $\alpha \to \infty$, the above equation goes to

$$2\alpha(m + r)\beta = (1 - B^{-1}\alpha^2)rF. \tag{31}$$

In a dimensional form,

$$k^2 + \frac{2(\nu_1 + \nu_2)b}{\sigma}k - \frac{\Delta\rho g}{\sigma} = 0. \tag{32}$$

Unlike Eq. (28), both liquid and vapor viscosities appear in the critical equation.

[8] and [9] provided the critical equation for viscous fluids with equal kinematic viscosities and semi-infinite thicknesses.

$$k^2 + \frac{4\nu b}{\sigma}k - \frac{\Delta\rho g}{\sigma} = 0. \tag{33}$$

One can see that Eq. (32) immediately reduces to (33) when $\nu = \nu_1 = \nu_2$. [8] claimed that the critical equation for a thin vapor layer was also given by Eq. (33). However, we found that there was error in deriving Eq. (33) for a thin vapor layer in [8] . If the error is corrected (omitted here), the critical equation is given by Eq. (28). Consequently, the present critical equations are verified.

Figure 3 shows variations of the most unstable wavelength λ_d with F and R. For a thin vapor layer (F is small), the phase change does not influence λ_d. To elucidate this behavior, the dispersion relation of Eq. (24) is approximated with regard to F and β. While the zeroth- and first-order terms are zero, the second- and higher-order terms are not zero. Therefore, for the dispersion relation to be asymptotically satisfied, the second-order term must be zero:

$$\frac{\partial^2 D}{\partial F^2}F^2 + 2\frac{\partial D}{\partial F}\frac{\partial D}{\partial \beta}F\beta + \frac{\partial^2 D}{\partial \beta^2}\beta^2 = 0. \tag{34}$$

Solving this for c, we obtain the following non-trivial solution:

$$c = -\underbrace{\frac{1}{rF}\frac{m^2(\cosh 2\alpha - 2\alpha^2 - 1) + m(\sinh 2\alpha - 2\alpha) + mr\sinh 2\alpha + 2rm\alpha + 2\alpha^2 r + r\cosh 2\alpha + r}{m^2(\cosh 2\alpha - 2\alpha^2 - 1) + 2m\sinh 2\alpha + \cosh 2\alpha + 2\alpha^2 + 1}\beta}_{\text{effect of phase change}}$$

$$+\underbrace{\frac{1 - B^{-1}\alpha^2}{2\alpha}\frac{m(\cosh 2\alpha - 2\alpha^2 - 1) + \sinh 2\alpha - 2\alpha}{m^2(\cosh 2\alpha - 2\alpha^2 - 1) + 2m\sinh 2\alpha + \cosh 2\alpha + 2\alpha^2 + 1}}_{\text{effect of buoyancy}} \tag{35}$$

The first term in the right-hand-side accounts for the effect of phase change. Since $\cosh 2\alpha - 2\alpha^2 - 1 > 0$ and $\sinh 2\alpha - 2\alpha > 0$ for $\alpha > 0$, the phase change always reduces the growth rate. The second term includes the effect of buoyancy and surface tension. This term has a destabilizing effect when $0 < \alpha < B^{1/2}$. For an extremely thin vapor layer ($\alpha \ll 1$), Eq. (35) can be further approximated as

$$c = -\frac{1}{F}\beta + \frac{1}{3}\alpha^2(1 - B^{-1}\alpha^2) + O(\alpha^3). \tag{36}$$

One can see that the critical wavenumber is affected by β, whereas the most unstable wavenumber remains unchanged as $\alpha_d = (B/2)^{1/2}$. On the other hand, for a thick vapor layer, the most unstable wavelength increases as the phase change increases, as shown in Fig. 3.
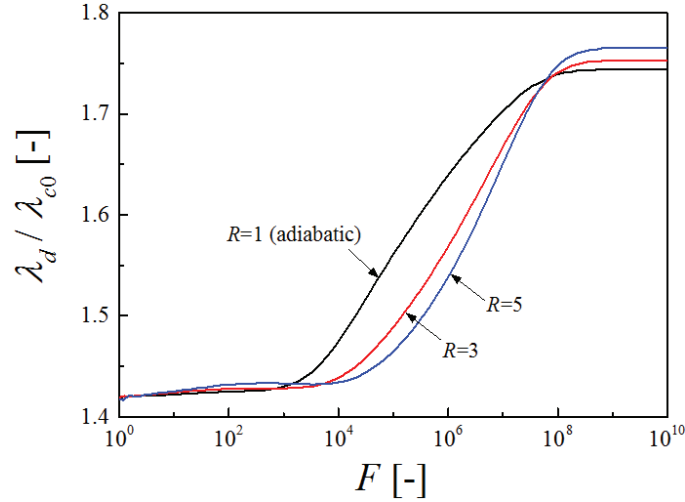

Figure 3. Effect of the evaporation on the most unstable wavelength

4. CONCLUSIONS

We have derived the dispersion relation for Rayleigh-Taylor instability of viscous fluids with heat and mass transfer. The vapor layer is initially in equilibrium with the liquid at the interface. The dispersion relation is given by Eq. (24). The phase change was shown to reduce the growth rate, i.e., stabilizing effect.

The critical equation was obtained as Eq. (26). Two extreme cases were considered. For a thin layer of vapor, the critical equation was given by

$$k^4 - \frac{\Delta\rho g}{\sigma}k^2 + \frac{3v_1 b}{\sigma\delta_1^3} = 0 .$$

There is a coupled effect of the vapor viscosity, phase change, and vapor layer thickness on the critical wavenumber. For a very thick layer of vapor, the critical equation was given by

$$k^2 + \frac{2(v_1 + v_2)b}{\sigma}k - \frac{\Delta\rho g}{\sigma} = 0 .$$

The effect of the vapor layer thickness is weakened and both liquid and vapor viscosities affect the critical wavenumber. The critical wavenumber decreases with the magnitude of phase change.

We also investigated the most unstable wavelength. For a thin layer of vapor, it was given by

$$\lambda_d = 2\pi\left(\frac{2\sigma}{\Delta\rho g}\right)^{1/2} ,$$

and is insensitive to the degree of phase change. However, for a thick layer of vapor, the most unstable wavelength was given by

$$\lambda_d > 2\pi\left(\frac{3\sigma}{\Delta\rho g}\right)^{1/2} ,$$

and it increases with the degree of phase change.

ACKNOWLEDGMENTS

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