ASSESSMENT OF THE BEST ESTIMATE THERMAL DESIGN METHOD USING THALES SUBCHANNEL CODE

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ABSTRACT

In order to maintain the safety of the reactor core, the minimum DNBR in the PWR core have to remain higher than the DNBR limit during Condition I and II events. Therefore, it is important to adequately determine the DNBR limit for the reactor safety and economic feasibility. To realistically evaluate the relationship among the uncertainties and reduce the conservatism resulting from the unknown phenomena, KEPCO NF introduces the Monte Carlo method for the modification and improvement of the existing thermal design method. For the best estimate thermal design method, various researches are conducted as follows. First, the random sampling methods are investigated to generate the Gaussian random numbers. Second, the normality test methods are reviewed to verify the normality of the generated random numbers. Third, the subchannel code, THALES, is used for the subchannel analysis. THALES is a COBRA family code developed by KEPCO NF. Through the subchannel analysis, the $DNBR_v$ distribution can be obtained. Forth, the $DNBR_v$ distribution is statistically combined with the uncertainties of the other parameters, e.g. CHF correlation etc. Then, the total DNBR distribution is generated and the DNBR limit can be determined to avoid DNB at a 95% probability at a 95% confidence level. Finally, through the example calculation, it is verified that this method produces reasonable results. If more studies are done, the best estimate thermal design method is useful to determine the DNBR limit.

KEYWORDS
DNBR, THALES, Monte-Carlo Method, DNBR Limit, Subchannel Analysis

1. INTRODUCTION

For the reactor safety, the fuel rods in the reactor core should not experience the CHF (Critical Heat Flux). In other words, the minimum DNBR (Departure from Nucleate Boiling Ratio) in the PWR (Pressurized-Water Reactor) core has to be higher than the DNBR limit during Condition I and II events. So, the nuclear power plants are always operated with the sufficient safety margin between the minimum DNBR and DNBR limit. In past years, the DNBR limit had been evaluated very conservatively, which causes the limitation of the core power uprating and increases the construction cost of the nuclear power plant. To reduce too large margin and operate the nuclear power plant economically, the DNBR limit needs to be assessed properly.

The DNBR limit is established based on Standard Review Plan 4.4 [1] of USNRC (U.S. Nuclear Regulatory Commission). According to Standard Review Plan 4.4, the acceptance criteria of the fuel design limit is described as follows: “One criterion provides assurance that there be at a 95-percent probability at the 95-percent confidence level that the hot fuel rod in the core does not experience a DNB (Departure from Nucleate Boiling) or transition conditions during normal operation or AOOs (Anticipated Operational Occurrences).” The following paragraph presents the combination of the uncertainties in the parameters. “Uncertainties in the values of process parameters (e.g. reactor power, coolant flow rate, core bypass flow, inlet temperature and pressure, nuclear and engineering hot channel factors), core design parameters, and calculation methods used in the assessment of the thermal margin should be treated with at least 95-percent probability at the 95-percent confidence level." and "Each uncertainty parameter should be identified as statistical or deterministic and should clearly describe the methodologies used to combine uncertainties."

The most important thing of the thermal design is the statistical combination of the uncertainties in the parameters. To achieve this, the Monte Carlo method is used at present. The Monte Carlo method helps realistically evaluate the relationship among the uncertainties and reduce the conservatism resulting from the unknown phenomena. The representative thermal design methods based on the Monte Carlo method are MSG (Méthode Statistique Généralisée) [2] and MTDP (Monte Carlo Thermal Design Procedure) [3] developed by Framatome and Belgatom, respectively.

For the best estimate evaluation of the uncertainties in the PWR core, KEPCO Nuclear Fuel (hereinafter KEPCO NF) has been developing the thermal design method on the basis of the Monte Carlo method. In Chapter 2, the major features of the best estimate thermal design method are described. The example calculation using this method is performed in Chapter 3.

2. DESCRIPTION OF THE BEST ESTIMATE THERMAL DESIGN METHOD

Fig. 1 shows a reason why the DNBR limit is needed simply. Since all the parameters that affect the DNB have the uncertainties with their own distribution, the DNBR consequently has the uncertainty, which leads to the occurrence of DNB. Therefore, the DNBR limit is reasonably determined to avoid the DNB at a 95-percent probability at a 95-percent confidence level. In this chapter, the main characteristics of the best estimate thermal design method being developed by KEPCO NF are described.

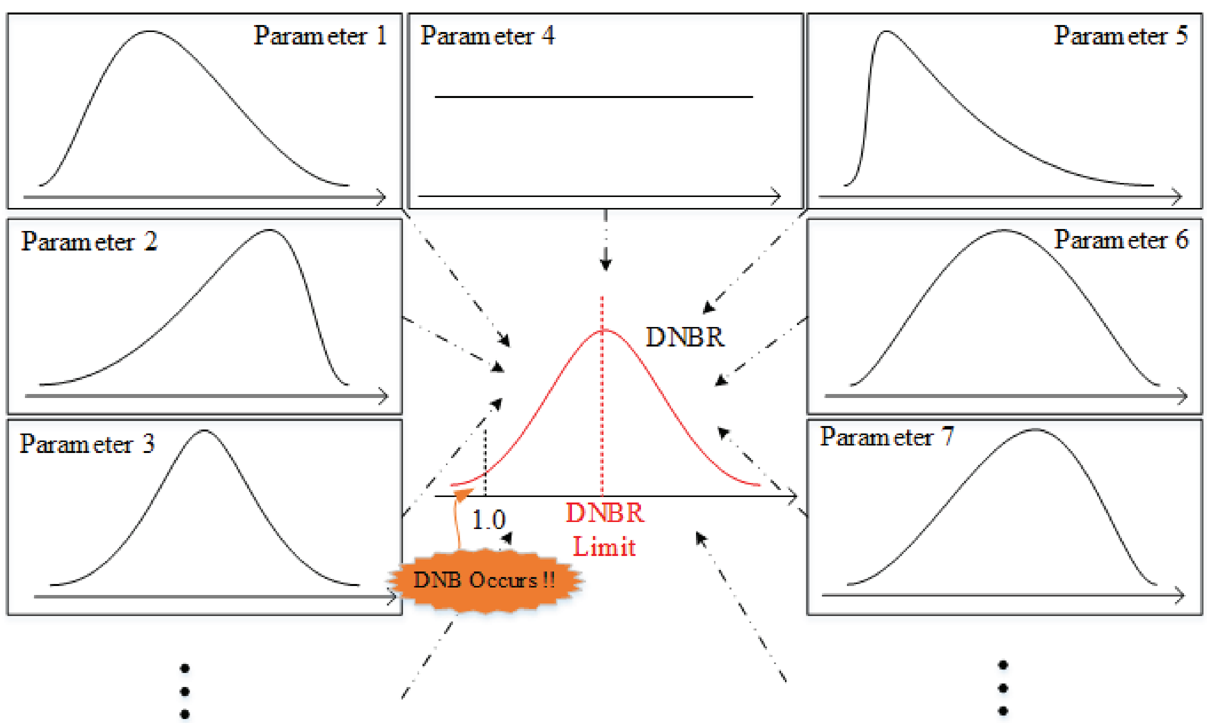

Figure 1. Brief Summary of the Best Estimate Thermal Design Method

2.1. Analysis Cases

It is not possible to evaluate the thermal performance of the core at all the operating conditions. Therefore, the representative operating conditions are selected, which are called the analysis cases. The analysis cases are divided into two categories – steady-state and transient analysis cases. Firstly, the steady-state analysis case is set in the range of the possible operating conditions during the normal operation of the nuclear power plant. The steady-state analysis case is generally a combination of the core power and system pressure. Secondly, the transient analysis case means the postulated limiting operating condition, which is chosen from the transient analysis that may occur during the operation of the reactor core.

2.2. Uncertainties of the Parameters

The parameters that affect the thermal performance of the reactor core have the uncertainties. Each parameter has its own magnitude and distribution. The uncertainties considered in the best estimate thermal design method can be classified into three types. The first type is the uncertainties of the core operating parameters, which is due to the measurement, instrumentation, and data processing, etc. The second type comes from the CHF (Critical Heat Flux) correlation. The CHF correlation has the uncertainty because the CHF correlation is obtained from the M/P distribution, where M/P is the ratio of the measured data (M) to the predicted data (P) from the subchannel code. The last type is the uncertainties of the subchannel and transient codes. They are imposed to compensate the prediction uncertainty of the code by the regulation authority in general.

2.3. Reviews of the Statistical Techniques for Generating the Gaussian Random Numbers

Since the thermal performance of the reactor core is affected by various parameters, it is too complicated to investigate the relations between them. Therefore, the Monte Carlo method is introduced to realistically evaluate the relationship among the uncertainties of the parameters. The Monte Carlo method needs many random numbers. Because most of the uncertainties in the parameters have normal distribution, various Gaussian random number methods were investigated. In this section, some of the most widely used Gaussian random number methods and normality test methods are reviewed.

2.3.1. Random Sampling Methods

There are plenty of the ways to generate the Gaussian random numbers. In this paper, three random sampling methods are reviewed. First, there is the CDF (Cumulative Distribution Function) inversion method, which uses the CDF directly. The form of the CDF, Φ(x), is as follow.

$$\Phi\left(x\right)=\int_{-\infty}^{x}\phi\left(x\right)dx=\frac{1}{2}\left[1.0+erf\left(\frac{x}{\sqrt{2}}\right)\right] \tag{1}$$

where ϕ(x) is the PDF (Probability Density Function). The form of the PDF with zero mean and unit variance is as follow.

$$\phi\left(x\right)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \tag{2}$$

Using Eq. 1, the CDF inversion method is expressed in Eq. 3.

$$x = \Phi^{-1}\left(u\right) \tag{3}$$

where u is the uniform random number in the interval (0,1) and x is the resultant Gaussian random number. Since the CDF is a function of the error function as shown in Eq. 1, this method consumes a lot of time. To supplement its weakness, the polynomial expression instead of the error function is used; however, the use of the polynomial expression has to be careful because the accuracy of the Gaussian random numbers depends on that of the polynomial expression.

The second method is the transformation method. The representative transformation method is Box-Muller method [4]. Box-Muller method transforms two independent uniform random numbers (u,v) in the interval (0,1) into two independent Gaussian random numbers (x,y) using Eq. 4.

$$x = \sqrt{-2\ln\left(u\right)} \cdot \sin\left(2\pi v\right), \quad \mathrm{y} = \sqrt{-2\ln\left(u\right)} \cdot \cos\left(2\pi v\right) \tag{4}$$

The third method reviewed in this paper is the rejection method. Of many rejection methods, polar method is introduced [5-6]. Polar method is a modification version of Box-Muller method, which changes two independent uniform numbers (u,v) in the interval (-1,1) into two independent Gaussian random numbers (x,y) using Eq. 5.

$$x = \sqrt{\frac{-2\ln\left(S\right)}{S}} \cdot u, \quad \mathrm{y} = \sqrt{\frac{-2\ln\left(S\right)}{S}} \cdot v \tag{5}$$

where,

$$S = R^2 = u^2 + v^2 < 1.0 \tag{6}$$

Polar method has the advantage of avoiding the trigonometric functions directly. Polar method can be schematized in Fig. 2.

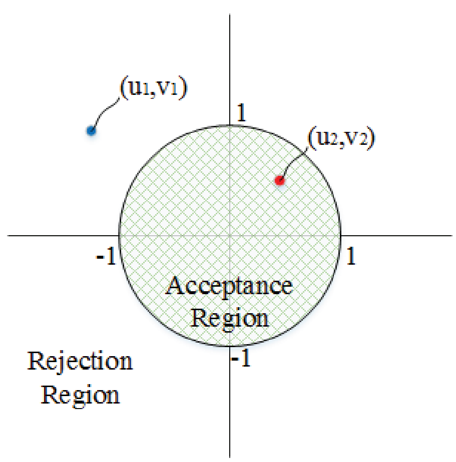

Figure 2. Schematic Diagram of the Polar Method

If (u,v) is located inside the circle (acceptance region), (x,y) is accepted as the Gaussian random numbers. Otherwise, (x,y) cannot be used. In other words, if S exceeds 1.0 or is equal to 0.0, the uniform random number (u,v) used ((u_1,v_1) in Fig.2) are rejected and two new independent uniform random number (u*,v*) are generated ((u_2,v_2) in Fig.2).

2.3.2. Normality Tests

For the use of the Gaussian random numbers generated in Section 2.3.1, the normality tests must be carried out. The normality test is based on two hypotheses as follows.

• Null hypothesis: the random numbers follow the normal distribution.
• Alternative hypothesis: the random numbers do not follow the normal distribution.

If the null hypothesis is accepted at the 5% significance level through the normality tests, it is concluded that the random numbers follow the normal distribution. Otherwise, the random numbers do not follow the normal distribution at the 5% significance level. The representative normality test techniques used in this paper are in Table I.

Table I. Representative normality test methods

Statistical Techniques	Normality Tests
Frequency Test	χ^2 test
Empirical Distribution Function (EDF) Statistics	Kolmogorov-Smirnov (K-S) Test
Correlation Test	Shapiro-Wilk Test
Moment Test	D'Agostino-Pearson Test

The χ^2 test is the statistical technique that uses the difference between the observed frequency and expected frequency at the each interval. Next, the K-S test is the normality test method that compares the difference between the empirical distribution function (EDF) and theoretical probability density function. In case of χ^2 test and K-S test, the critical value is determined based on the degree of freedom and significance level. If the normality test results are less than the critical value, the null hypothesis is accepted.

Shapiro-Wilk test [7] uses the ratio of the variance of the normally distributed weighted least square estimates and that of the random numbers generated in Section 2.3.1. If Shapiro-Wilk test is conducted, the test statistics W bounded by 0.0 and 1.0 is generated. Then, W is changed into p-value for the normality test. If p-value is larger than 0.05 (5% significance level), the random numbers is assumed to be normally distributed.

Finally, there is the moment test. This normality tests are used for sample moments, e.g. kurtosis and skewness. The representative moment test is the D'Agostino-Pearson test [8]. D'Agostino-Pearson proposed the omnibus test that combines skewness ($\sqrt{b_1}$) and kurtosis (b_2) in Eq. 7. The K^2 has approximately a χ^2 distribution, with two degree of freedom when the population is normally distributed.

$$K^2 = Z^2\left(\sqrt{b_1}\right) + Z^2\left(b_2\right) \tag{7}$$

2.4. Subchannel Analysis

The effect of the core operating parameters mentioned in Section 2.2 is evaluated through the subchannel analysis. In the best estimate thermal design method, the subchannel code THALES (Thermal Hydraulic AnaLyzer for Enhanced Simulation of core) is used. [9-10] THALES was developed by KEPCO NF for the thermal-hydraulic design in the PWR core. The control volume used in THALES is shown in Fig. 3. THALES is a COBRA family code [11-12] and on the basis of the single-stage core analysis model [13].

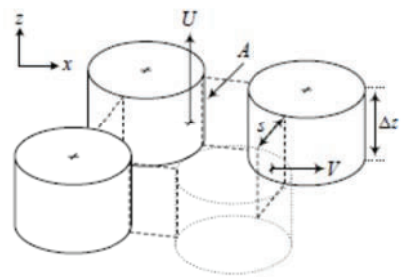

Figure 3. Control Volume of THALES

Many subchannel analyses are conducted on the basis of the core operating parameters which are randomized using the random sampling method in Section 2.3, and then the DNBR distribution ($DNBR_v$) is obtained as shown in Fig. 4. The $DNBR_v$ distribution will be combined with the other uncertainties in next step.

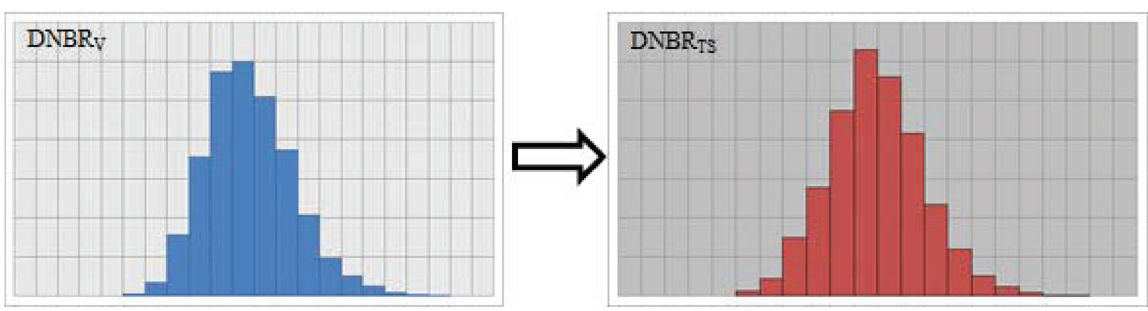

Figure 4. Histograms of $DNBR_v$ and $DNBR_{TS}$

2.5. Total DNBR Distribution

The $DNBR_v$ distribution considers the effect of the operating parameters only. The uncertainties of the CHF correlation, subchannel code, and transient code remain to obtain the DNBR limit. The uncertainty of the CHF correlation is included because the CHF correlation is used when the DNBR is predicted by the subchannel code. The uncertainty of the CHF correlation stems from the M/P distribution. Since the M/P distribution is obtained from the experiments and subchannel analyses, its mean and standard deviation are calculated from the finite samples. So, the uncertainty of the CHF correlation is estimated as the population mean and standard deviation with 95% confidence level. The uncertainty of the CHF correlation is randomized and it corrects each of the $DNBR_v$ distribution obtained in Section 2.4. Then, $DNBR_{v'}$ is obtained.

Finally, the uncertainties of the subchannel code and transient code are reflected, which are randomized using the random sampling methods in Section 2.3.1. The subchannel code and transient code uncertainties are also considered the each of the $DNBR_{v'}$, and then the total DNBR distribution ($DNBR_{TS}$) are obtained finally as shown in Fig. 4.

2.6. Determination of the DNBR Limit

Since the $DNBR_{TS}$ distribution is derived from the finite samples, it has the sample mean ($\bar{x}_{TS}$) and standard deviation (S_{TS}). To determine the DNBR limit, the population mean (μ_T) and standard deviation (σ_T) are needed. The population mean and standard deviation are estimated using Eq. 8 and 9 with the (1-α) confidence level.

$$\mu_T = \bar{x}_{TS} + t_{f,1-\alpha} \frac{S_{TS}}{\sqrt{N}} \tag{8}$$

$$\sigma_T^2 = \frac{f \cdot S_{TS}^2}{\chi_{f,1-\alpha}^2} \tag{9}$$

where N is the number of the random numbers and f is the degree of the freedom (N-1). $T_{f,1-\alpha}$ and $\chi^2_{f,1-\alpha}$ are the t-distribution and Chi-square distribution, respectively. From the $DNBR_T$ distribution as shown in Fig. 5, the DNBR limit is obtained to prevent the DNB at a 95/95 criterion in Eq. 10. However, the distribution-free tolerance limit is used if the $DNBR_{TS}$ distribution does not follow the normal distribution in (1-α) confidence level [14-15].

$$\text{DNBR Limit} = 1.0 + 1.645 \times \sigma_T \tag{10}$$

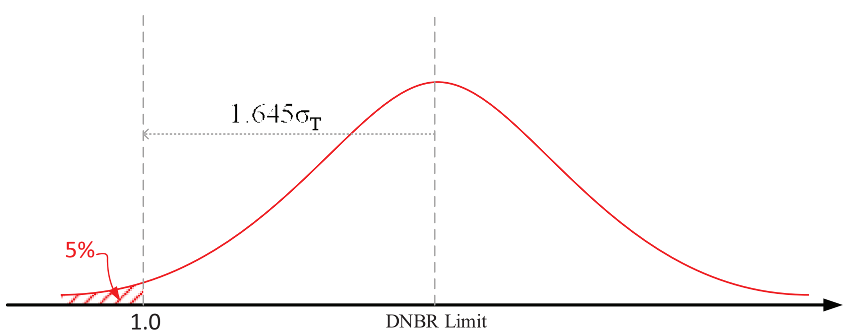

Figure 5. Schematic Diagram for Determining the DNBR Limit

3. EXAMPLE CALCULATIONS

In this chapter, the example calculation is presented to verify this thermal design method. First, the analysis cases are determined. The steady-state analysis cases contain the core operating limit range, which form the combination of the core power and pressure in general. In this paper, four representative steady-state analysis cases are chosen in below.

• Case 1: High Power and High Pressure
• Case 2: Low Power and High Pressure

• Case 3: High Power and Low Pressure
• Case 4: Low Power and Low Pressure

In addition, one transient analysis case is added for the best estimate thermal design method in below.

• Case 5: Loss of Flow

Next, the magnitude and distribution type of the uncertainties are defined in Table II.

Table II. Magnitude and distribution type of uncertainties in the core operating parameters

Parameters	Magnitude	Distribution Type
Reactor Power	2.0 %	2-Sided Normal
Inlet Temperature	6 ^{o}F	2-Sided Normal
Pressure	50 psi	2-Sided Normal
Coolant Flow Rate	4.2 %	2-Sided Normal
Core Bypass Flow	1.5 %	Uniform
Nuclear Hot Channel Factor	4.0 %	1-Sided Normal
Engineering Hot Channel Factor	3.0 %	1-Sided Normal

Using the nominal values of the analysis case and uncertainties of the core operating parameters, the core operating parameters are randomized. In this example, 3,000 random numbers generated from the polar method are used. For example, Fig. 6 shows the histograms of the randomized core operating parameters.

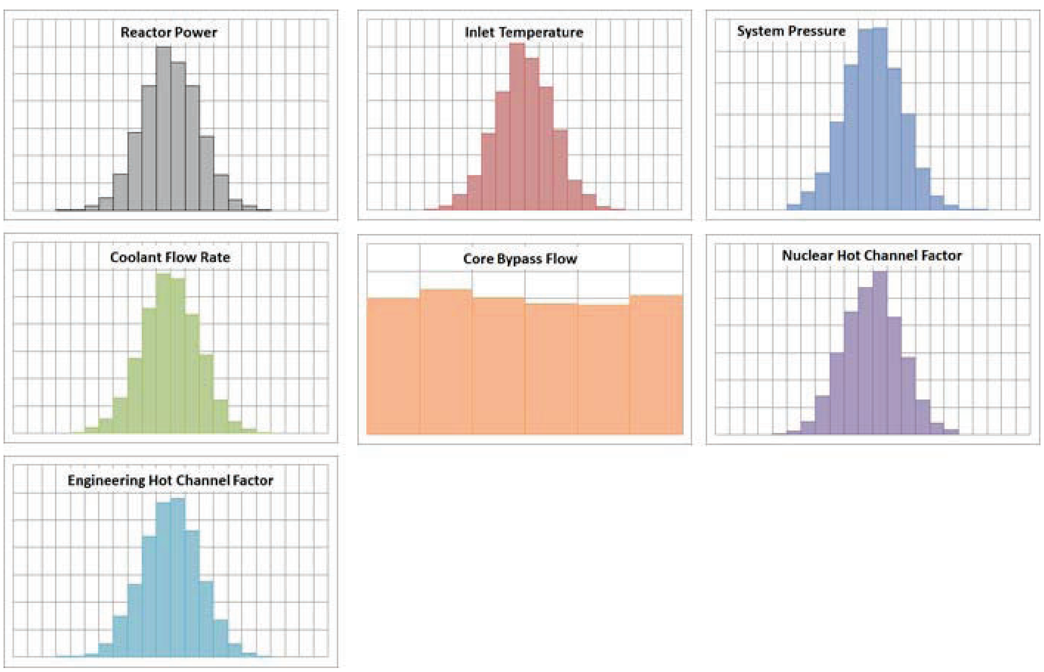

Figure 6. Histograms of the Randomized Core Operating Parameters

To verify the normality of the random numbers, the normality tests are conducted. For example, the normality test results of the core operating parameters used in the analysis case 1 are presented in Table III. The critical values are given in parenthesis in Table III. As shown in Table III, all the core operating parameters pass the normality tests, which mean that all the core operating parameters randomized follow the normal distribution at the 5% significance level.

Table III. Normality test results of the core operating parameters

Normality Test Methods	χ^2 Test (32.6706)	K-S Test (0.0248)	Shapiro-Wilk Test (0.05)	D'Agostino-Pearson Test (5.9915)
Reactor Power	9.2675	0.0111	0.3800	0.3458
Inlet Temperature	13.8202	0.0069	0.3989	0.3780
Pressure	13.7587	0.0082	0.3916	1.5559
Coolant Flow Rate	7.7841	0.0097	0.3504	0.5508
Nuclear Hot Channel Factor	7.9425	0.0121	0.3215	2.3545
Engineering Hot Channel Factor	14.2948	0.0096	0.3829	1.0842

Since the normality tests are passed, the randomized core operating parameters can be used. For the subchannel analysis, the randomized core operating parameters are combined each other and then 3,000 core operating parameter sets are generated. Based on these sets, the subchannel analyses are conducted using subchannel code THALES and $DNBR_v$ distribution is obtained. The histograms of the $DNBR_v$ distribution are shown in Fig. 7.

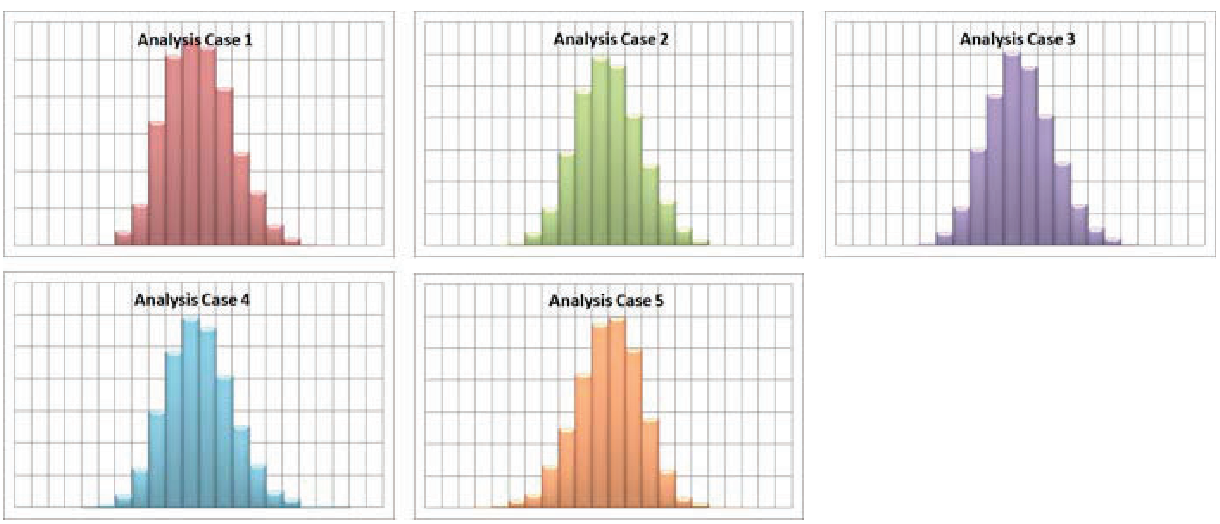

Figure 7. Histograms of $DNBR_v$

In this stage, the uncertainties of the CHF correlation, subchannel code, and transient code are reflected in the $DNBR_v$ distribution, and then $DNBR_{TS}$ distribution is obtained. Fig. 8 shows the histograms of $DNBR_{TS}$.

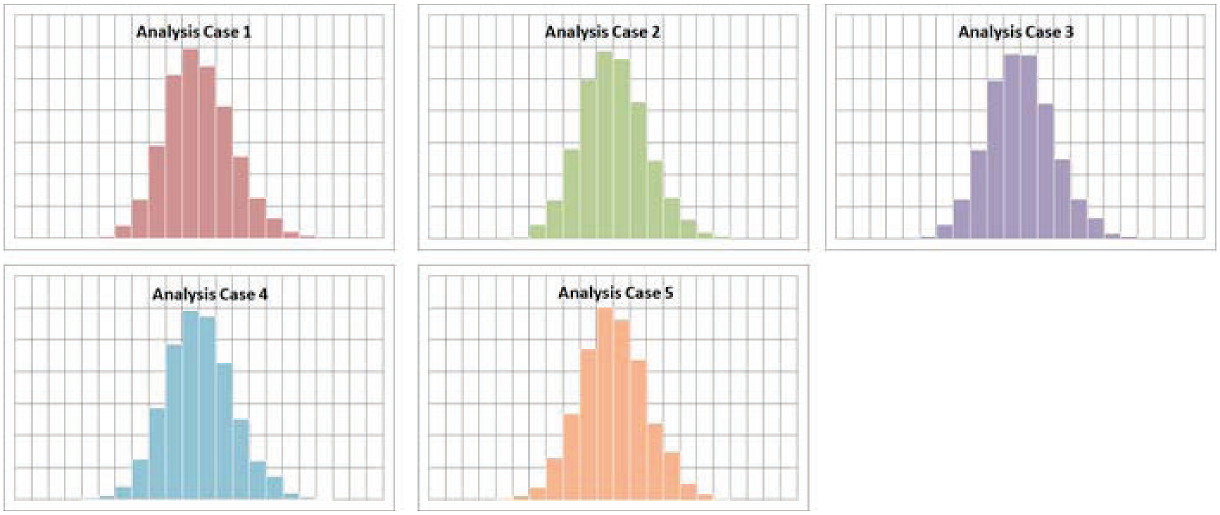

Figure 8. Histograms of $DNBR_{TS}$

From the $DNBR_{TS}$ distribution, the sample mean ($\bar{x}_{TS}$) and standard deviation (S_{TS}) are calculated. Substituting $\bar{x}_{TS}$ and S_{TS} in Eq. 8 and 9, μ_T and σ_T are estimated with 95% confidence level. Using σ_T and Eq. 10, the DNBR limit can be calculated. The calculation results of σ_T and DNBR limit in all the analysis cases are summarized in Table IV.

Table IV. σ_T and DNBR limit in all the analysis cases

Analysis Cases	σ_T	DNBR Limit
Case 1	0.12509	1.2058
Case 2	0.13288	1.2186*
Case 3	0.11826	1.1945
Case 4	0.13206	1.2172
Case 5	0.12141	1.1997

* : limiting case

In case of the example calculation, the DNBR limit is set to 1.2186, which assures that a DNB in the hot fuel rod do not occur at least 95% probability at a 95% confidence level during Condition I and II events.

4. CONCLUSIONS

For the best estimate evaluation of the uncertainties in the parameters, KEPCO NF has been developing the thermal design method based on the Monte Carlo method. To develop this method, various random sampling methods and normality test methods are reviewed and applied. The random sampling methods make the core operating parameters randomized, which can be used only if the normality tests are passed. Second, the subchannel analysis code THALES is used to obtain the DNBR. Finally, a method to statistically combine the uncertainties is investigated. To verify this method, the example calculation is carried out, which is shown that this method reasonably produces the DNBR limit.

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